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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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THEORY OF SIMILARITY AND MODELS

By Benjamin F. Groat,* M. Am. Soc. C. E.

Synopsis

The paper aims to present a complete, concise treatise on the theory of similarity and models. Newton's theorem, which is not generally understood, is shown to cover every possible type of dynamic model. A misconception which has been unduly propagated, is disclosed and traced to its source. It is shown that models can be built to specification, as to materials and their properties, so as to perform with theoretical exactness. It is suggested that materials, including fluids, should be sought, made, or arranged, so as to adapt them especially for service in models, thus opening a new field in engineering. The theory is based upon data given in a previous paper by the writer.†

Introduction

Every engineer should know and understand that a model, constructed in accordance with Newton's theory of similarity, is not only the means for reproducing the complexities of flow, or other mechanical action, on a small scale, but that such model may be made the best means of designing works of great magnitude. In the present state of knowledge, a properly constructed and properly tested model will answer the difficult questions of hydrodynamics much more quickly and accurately than the most profound mathematical analysis.

Engineers and other groups of society incline too little toward making the most of past experience. It is about 250 years since Newton enunciated the principles of similarity and the theory of models; yet it is only during the last very few years that engineers have made any use of models, as a reliable means for determining a design, especially in hydraulics.

Note.—Written discussion on this paper will be closed in **February, 1931**, *Proceedings*.

* Cons. Engr., Brookline, Mass.

† "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138.

The cause of this inertia in the advancement and dissemination of science and truth, is the tendency to place a blind reliance upon the authority who makes a statement, instead of basing belief on analysis of what is said. Civilization should be standardized on the basis of reason; it should not be allowed to develop along a mere line of natural social evolution. Were it not that Lincoln said it, the slogan, "Government of the people, for the people, and by the people," would be a forgotten generalization to-day. It is "harped" upon because a great man said it, but, in reality, too little has been done toward a sufficient application of this, one of the greatest generalizations of all time. Is not the Engineering Profession best prepared to overcome inertia of every kind all along the front?

Notation

The following notation will be used consistently in this paper and its discussion:

$\propto$ = varies as.
g = acceleration due to gravity; also, acceleration in model generally.
V = velocity in prototype, or ratio of homologous velocities according to context.
D = diameter, or ratio of diameters, according to context.
l = linear magnitude in model.
p = pressure.
μ = absolute coefficient of viscosity, usually in centimeter-gramme-second units.
ρ = density in model, or ratio of homologues, according to context.
P = property of a material, or ratio of such. In Appendix I, it is ratio of homologous pressures.
T = time corresponding to a. (Stokes.)
T = time dimension, or ratio of homologous times. Sometimes as $[T]$.
W = specific weight in prototype, or ratio of homologues.
L = linear dimension, or ratio of such homologues. Sometimes as $[L]$.
F = force dimension, or ratio of such homologues. Sometimes as $[F]$.
$\nu = \frac{\mu}{\rho}$ = kinematic coefficient of viscosity of the fluid.
R = scale ratio.
G = acceleration in prototype, or acceleration ratio.
N = kinematic coefficient of viscosity of the fluid in the prototype.
E = ratio of homologous elastic moduli.
k = elastic spring constant.
Δ = density in prototype.
M = mass ratio.
M = absolute coefficient of viscosity in prototype, or ratio of homologues.
f = sign of functionality.
P = pressure of gas. (See, also, previous definitions.)
c = gas constant in Appendix I.
w = specific weight of a gas.
θ = absolute temperature, in degrees centigrade.
K = a constant. Reynolds' number in Equation (9).
ϕ, ϕ' = signs of functionality in Equations (20) and (21).

Historic Background

History may now be recited to show how the theory of models and similarity made its appearance, and what the situation is at present (1930) relative to

the use of models. It was more than 100 years after the greatest natural philosopher, Sir Isaac Newton, adduced the theory—in fact, the whole idea—expressed in true scientific simplicity, before any use was made of it. Most individuals fail to remember that the entire theory of terrestrial mechanics was given by Newton in three simple rules, or "laws," which can be written in about as many lines. The entire basis of celestial mechanics was delivered by that same philosopher in a statement of two or three more brief lines. Newton propounded the theory of models in a single paragraph, and nothing has since been added to any of these theories, other than their natural consequences.

In his "Principia Mathematica" (original in Latin), Newton published the theory of similarity* as follows:

"Suppose two similar systems of bodies consisting of an equal number of particles, and let the correspondent particles be similar and proportional, each in one system to each in the other, and have a like situation among themselves, and the same given ratio of density to each other; and let them begin to move among themselves in proportional times, and with like motions (that is, those in one system among one another, and those in the other system among one another). And if the particles that are in the same system do not touch one another, except in the moments of reflexion; nor attract nor repel each other, except with accelerative forces, that are as the diameters of the correspondent particles inversely, and the squares of the velocities directly, I say that the particles of those systems will continue to move among themselves with like motions and in proportional times."

The term, "accelerative force," means the intensity of a force per unit mass acted upon by the force. The most common example is the acceleration of gravity, g, which is the acceleration of a freely falling body at the earth's surface. If this is multiplied by the measure of the mass acted upon, the result is the total force acting upon the mass.

Hence, Newton's Law of Similarity supplies the fundamental equation of similitude, namely,

$$g = \frac{V^2}{D} \qquad (1)$$

in which, the symbols relate to ratios of homologues.

Two systems of forces in space are dynamically similar if the lines of action of the forces (produced if necessary) form similar geometrical figures in space, while the measures of the two individuals of every pair of homologous forces in the two systems are in a fixed ratio (called the "force ratio of the homology"). Thus it is clear that proportionality of homologous forces, and geometrical similarity of the figures formed by the systems of lines representing their magnitudes and loci, are both involved in the idea of dynamical similarity.

Every engineer is familiar with the force diagram and the space diagram of graphical statics. Proportionality of homologous forces in dynamically similar systems corresponds to similar force polygons, while similarity of applications of forces to masses, similarly situated in two similar systems, corresponds to similar space polygons. Thus, two geometrically similar framed

* Book II, Proposition XXXII, Theorem XXVI.

structures, bearing proportional loads similarly distributed, would sustain similar internal stresses, any two homologues of which would be in the force ratio. One such frame, so analyzed, would be the solution of all such similar frames bearing similar systems of loading.

Correspondingly, the term, "mass similarity," may be used to express the idea that two systems of masses occupy space in such configurations that the measures of homologous masses are in a fixed ratio called the "mass ratio of the homology", while the loci of the homologous masses are similarly situated in the two systems.

The complete notion of mechanical similarity embraces the idea of "similar motions," in addition to the ideas of dynamical similarity and mass similarity. Similar motions require that the paths described by homologous masses be geometrically similar, and that the velocities of homologous masses be in a fixed ratio, which may be called the "velocity ratio."*

Newton's law simply states the conditions which are necessary and sufficient to establish and maintain dynamic similarity, including geometrical similarity of both form and motion. It is to be emphasized that any one of these conditions must be satisfied in order to have similarity, but any one of them alone would not be sufficient without the others. For example, it is necessary to establish initial similarity, or the theorem would not apply at all. This means, furthermore (taking a practical example), that if a model of a section of a river is built, it is necessary that the water enter and leave the model with motions and levels exactly similar to the homologues in the actual river. This condition is as necessary as any of the others. Observe, in particular, that accelerative forces must be proportional to the squares of velocities directly and to the linear dimensions (diameters, for example) inversely. This part of the conditions relates especially to similar motions. However, the theorem does not require that accelerative forces be constant throughout any given system. Indeed, they may vary discontinuously from point to point. They may be concentrated at a point, for example. The theorem does require that the ratios of all pairs of homologous accelerations be the same.

This relation between accelerative forces and velocities has been erroneously called "Froude's law," because of his use of it in the celebrated tests of "skin friction" on boards (sides of ships), and especially in his tests of ship models to determine the wave-making resistances to their motion through the water. Froude's law has been written in the form of a dimensionless number, $\left(\frac{V^2}{l\,g}\right)$. The statement that it is constant is nothing more or less than Newton's law, $g \propto \frac{V^2}{D}$ (see Equation (1)). Note that l may be inserted in the formula in the place of D, since the ratio of similar dimensions is constant, regardless of what dimension is used.

Newton's law should be distinguished from the law of skin friction, which states that resistances per unit area vary as the squares of velocities. Equation (1) expresses the relation between model and prototype, while the fric-

* "Ice Diversion for St. Lawrence River Power Co.," by B. F. Groat, M. Am. Soc. C. E., *The Canadian Engineer*, November 25, 1920.

tional resistances may follow another law; for example, they may vary directly as the velocity. This last relationship is quite consistent with Equation (1). When the common law of friction (V^2 law) holds, the model may be of any size, but when friction varies with a power of the velocity different from 2, the scale of the model ceases to be arbitrary.*

Newton's law of similarity was stated so as to cover models in general, independent of size, shape, and materials. To make this sweeping statement, its originator was careful not to permit the possibility of unknown reactions between the moving bodies. Consequently, he excluded every contactual influence except instantaneous contact of the moving particles, not because of any fundamental necessity for such restriction, but to emphasize that the forces of dynamic similarity must be proportioned so as to cause, or develop, the accelerations specified by him in the law, as he stated it.

It was not until 1850 that Stokes,† in the course of his investigations, noticed certain important mathematical relations involved in the theory of similarity and models. Through a slight oversight, however, he rendered an account of similarity involving viscosity which has led a number of investigators into the same error.

As its title shows, Stokes' paper was primarily concerned with viscosity. This is a special type of force which Newton wisely did not mention specifically, although his theorem, or law, does include any force which can be made to conform to it.

Stokes was an able pioneer investigator in hydrodynamics, a subject full of fundamental mathematical forms devised by him. He essayed to study the laws of similarity in viscous fluids, since it was necessary to ascertain the effect of changing the sizes, shapes, and conditions of bodies moving in a viscous fluid, such as air. He showed that the sizes of bodies did not change the mathematical treatment. In other words, the same differential equations would represent each of two, or more, similar systems.

His paper includes one of the most remarkable contributions touching upon similarity and models since Newton's great generalization. It covers the entire question of viscosity; but its author did not make his account of similarity in the presence of viscous fluids as complete as this part of the investigation can be made, as will appear after considering his very simple equations relative to similarity in the presence of viscous fluids.† This part of Stokes' paper requires some knowledge of coefficients of viscosity, which has been previously explained by the writer.‡

Analysis of Stokes' Remarks Upon Similarity

Consider any number of similar systems, composed of similar solids oscillating in a similar manner in different fluids or in the same fluid. Let

* This follows from the writer's article in *Engineering Record*, September 25, 1915, p. 377.

† "On the Effect of Internal Friction of Fluids on the Motion of Pendulums," *Transactions*, Cambridge Philosophical Soc., 1850, p. 19, Section 5.

‡ In his discussion of the paper entitled, "Experiments on Discharge over Spillways and Models, Keokuk Dam," by Floyd A. Nagler and Albion Davis, Members, Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., Papers and Discussions, October, 1929, p. 2207, *et seq.*

a, a', ..., be homologous lines in the different systems; T, T', ..., corresponding times, such, for example, as the times of oscillation from rest to rest. Let x, y, and z, be measured from similarly situated origins, and in corresponding directions, and t, from corresponding epochs, such, for example, as the commencements of oscillations when the systems are beginning to move from a given side of the mean position.

If any dynamical equations (including differential equations, of course) are satisfied for one system they will be satisfied for all the systems provided $u \propto v \propto w$; $x \propto y \propto z$; and $p \propto \frac{\mu u}{x} \propto \frac{\rho u x}{t}$, when passing from system to system. Stokes did not say that $p \propto \frac{\rho u x}{t}$ is the statement of Newton's law. The variations, $x \propto y \propto z$, merely signify that similarly situated points must be compared in inferring, from the circumstance that equations are satisfied for one system, that they will be satisfied for all the systems. If c, c', ... , are the maximum displacements of similarly situated points of the fluids, then $u \propto \frac{c}{T}$; $x \propto a$; and $t \propto T$.

Quoting Stokes,* "the sole condition to be satisfied, in addition to that of geometrical similarity, in order that the systems should be dynamically similar, becomes" $\frac{a^2}{T} \propto \frac{\mu}{\rho}$, or $\frac{a^2}{T} \propto \nu$. This condition being satisfied, similar motions will take place in the different systems; therefore, $p \propto \frac{\rho a c}{T^2}$.

It follows from equations that might be written down by symmetry, that the pressures, such as normal or tangential, vary in the same manner as p; whence it appears, from the relation, $p \propto \frac{\rho a c}{T^2}$, that the resultant or resultants of the pressures of the fluids on the solids, acting along similarly situated lines, which vary as $p a^2$, vary as ρa^3 and $\frac{c}{T^2}$ conjointly. In other words, these resultants in two similar systems are to one another in a ratio compounded of the ratio of the masses of fluid displaced, and of the ratio of the maximum accelerating effective forces belonging to similarly situated points in the solids.

Stokes' relation, $u \propto v \propto w$, means simply that (if capitals relate to the first system and minuscules to the second),

$$\frac{U}{u} = \frac{V}{v} = \frac{W}{w}$$

or, as Stokes writes it, $u \propto v \propto w$, in which the u's relate to x components of velocity, and the v's and w's, respectively, to the y and z components. The simplest way to see this is to note generally that the "dimensions" of velocity are, in all cases, $\frac{\text{distance}}{\text{time}} = \frac{L}{T}$. Thus, if the two systems are geometrically

* "On the Effect of Internal Friction of Fluids on the Motion of Pendulums," *Transactions*, Cambridge Philosophical Soc., 1850, p. 19, Section 5.

similar, and the times of motion over homologous distances are proportional, $u \propto v \propto w$ is satisfied for all the motions in two similar systems; that is, for all the homologous motions.

This is a general truth. If any two or more quantities have the same "dimensions" in F, L, and T, then, if every ratio of two homologues measured in a given simple dimension, L, F, or T, is in a given ratio which depends upon the type of the dimension, the quantities themselves are also in a determinate ratio, and that ratio is found by calculating the product of the known powers of the fixed ratios of their dimensions. Thus, the ratio, $\frac{V}{v}$, equals the product, $\left(\frac{L}{l}\right) \times \left(\frac{T}{t}\right)^{-1}$, and it will be constant from molecule to molecule if each of the factors is constant. Hence, one may substitute the sign of equality for the sign of variation when the symbols relate to ratios of homologues, as will frequently be the case hereafter.

In passing, it is well to call attention to the fact that confusion of ideas results from two distinct uses of the word, "dimension." One use relates to architectural dimensions, so familiar to all engineers; the other, to "dimensions of units," which is much less common. The first use relates to a measure in terms of a unit; the latter to the nature of the unit. Both the magnitude and its unit have the same dimensions (latter use). The measure of a magnitude is, therefore, a pure number (no dimensions). The measure of a fixed magnitude varies inversely as the magnitude of its unit, that is, of the unit employed in the measure. The complete specification of an architectural dimension consists of a measure followed by a specification of the unit, as 200 ft. Correspondingly, N ft.^{2}* names the unit as 1 sq. ft.; $N\frac{\text{ft.}^{*}}{\text{sec.}}$ names the unit as 1 ft. per sec.; etc. The dimensions (latter use) just specified are, respectively, feet, square feet, and feet per second. To avoid naming the unit along with a specification of its dimensions (latter use), the following forms are used, respectively, for the three cases mentioned, L, L^2 and $\frac{L}{T}$, or, $[L]$, $[L^2]$, and $\left[\frac{L}{T}\right]$. It is unfortunate that the word has been used in the latter sense, because architectural dimensions are measured in terms of units of certain dimensions. Thus, two different meanings of the word may occur in the same sentence.

If the three fundamental units, for measurements in the model, are so chosen as to bear the same respective magnitudinal ratios to their homologues as do corresponding homologous magnitudes, then the measurement of any quantity in the model will be precisely the same numerically, as in the prototype. One-twenty-fourth full size gives measures equal to those in full size when the scale is "$\frac{1}{2}$ in.=1 ft." Homologous ratios are merely cases of "con-

* This is the correct form in which the unit should be specified; it shows just how the unit is constituted. The notation is now universal when the nature of the unit is involved in the analysis.

version factors." In the example just mentioned the conversion factor is unity, by reason of the choice of the model unit.

To illustrate the nature of dimensions, take Stokes' requirement that $p \propto \frac{\mu u}{x} \propto \frac{\rho u x}{t}$. The respective dimensions of the individual quantities are:

$$p = \frac{F}{L^2} \quad \ldots\ldots (2)$$

$$\mu = \frac{F T}{L^2} \quad \ldots\ldots (3)$$

$$u = \frac{L}{T} \quad \ldots\ldots (4)$$

$$x = L \quad \ldots\ldots (5)$$

$$\rho = \frac{F T^2}{L^4} = \frac{M}{L^3} \quad \ldots\ldots (6)$$

$$t = T \quad \ldots\ldots (7)$$

$$\nu = \frac{\mu}{\rho} = \frac{L^2}{T} \quad \ldots\ldots (8)$$

Supplying these dimensions to all the terms, it may be shown that each one has the dimensions of pressure; that is, $p \propto \frac{F}{L^2}$, if L, F, and T are considered the fundamental dimensions of units, length, force, and time. Of course, it would have been found that all the terms were of the same dimensions, had mass been utilized as a fundamental unit, instead of force. Therefore, all that is required to insure these equal proportions is that forces be proportional to the areas upon which they act, the dimensions of area being L^2. This applies to shearing stress (viscosity) as well as to normal pressure.

It is clear generally, without further discussion, that since all the terms of any equation must necessarily have the same dimensions (apples cannot be added to peaches, so to speak), any equations expressing relations between homologous quantities in two similar systems will be the same for either system, except for a common constant factor in all its terms, which factor, of course, may be removed. If homologous units are suitably chosen the factor will be unity, as previously explained.

The relation, $\frac{a^2}{T} \propto \nu$, is the principal one for present purposes, since it imposes a new condition resulting from the introduction of viscosity into the calculation. It may be written in the form:

$$\frac{V L}{\nu} = K \quad \ldots\ldots (9)$$

in which, the constant, K, is a dimensionless number since $V = \frac{L}{T}$, and ν is defined as the ratio of μ to ρ, which is the kinematical coefficient of viscosity (see Equation (8)).

Of course, the constancy of Equation (9) is maintained only for homologous values in similar systems, as model and prototype. It expresses the well-known Reynolds' number, which, as it appears, was first recognized by Stokes; in fact, it was defined by Stokes, before Reynolds made any use of it. It would seem more appropriate to call it "Stokes' number." In this relation, the real debt of science to Reynolds is for his discovery of "turbulence," as compared to stream-line flow, and for his additional discovery of the "critical velocity" and its relation to Stokes' number as the criterion of the existence of turbulence.

The reason why Equation (9) (or its equivalent, Equation (10)) puts a new limitation on models is that the introduction of viscosity demands a new, and additional, relation (Equation (9) or Equation (10)) between two of the three fundamental ratios of the homology, L, F, and T, or L, M, and T, if preferred. These ratios are considered fundamental because they relate to the three units which are chosen as the "fundamental" units of the system of measures involved. Briefly, viscous fluids in similar systems require that,

$$\nu = \frac{\mu}{\rho} = \frac{L^2}{T} = \frac{a^2}{T} \ldots\ldots, \text{etc} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The symbol, ν is the kinematic coefficient of viscosity. It is clear why this coefficient is called "kinematical," since, in finding the ratio of μ to ρ (see Equation (10)) the dynamical dimension, "force" (or "mass," when mass is one of the three "fundamental" units, or dimensions), is cancelled, leaving the ratio dependent upon "length" and "time" only, and, therefore, "kinematical."

While the writer has the greatest admiration for Stokes' valuable analysis, he can find no justification for the statement* that Equation (10) as derived by Stokes, is "the sole condition to be satisfied, in addition to that of geometrical similarity, in order that the systems should be dynamically similar."

If Stokes had said: "It is the sole condition to be satisfied in addition to those of geometrical and dynamical similarity in the absence of viscous fluids, in order that the systems shall remain geometrically and dynamically similar after introducing viscous fluids," there could have been no reasonable objection. His analysis conforms exactly with this statement, and it is probably what he intended to say—"geometrical similarity" doubtless including geometrically similar motions, which latter condition demands dynamical similarity.

In other words, Stokes' systems were dynamically and geometrically similar, in accord with Newton's law $\left(g \propto \frac{V^2}{D}\right)$. This condition is also expressed in Equation (12), before introducing viscous fluids, or, what is the same thing, before causing the fluids to become viscous. His principal equations (which need not be discussed here) show this, as they correctly determine geometrically and dynamically similar systems without the terms involving viscosity, and without those involving "extraneous forces" (gravity). Introducing viscosity may, or may not, change ratios of homologues, but the simi-

* *Transactions*, Cambridge Philosophical Soc., 1850, p. 19, Section 5.

larity continues, provided the conditions of Newton's theorem (law) are imposed throughout the entire time.

Consequently, Equations (9) and (10), are not essentially criteria of dynamic similarity at all, but are merely necessary conditions that must be satisfied by any viscous fluids in the systems. They are conditions appurtenant to viscosity, not to similarity.

Were this a matter of opinion there would be little hope of interesting the engineer and analyst; but it is not a matter of opinion. It is a matter of mere mathematical relations, and it is the writer's purpose to show that, although these relations were plainly visible in Stokes' paper, they were overlooked by him in giving an account of his investigation. This was doubtless due to his greater concern with the more intricate mathematical analysis of the pendulum problems in the remainder of his paper, which need not be examined here.

Formal Theory of Similarity

Let two systems be constructed with the requirement that they shall be merely geometrically similar. Clearly, the sole requirement for this is that homologous linear dimensions be proportional. Stokes writes this in the form, $x \propto y \propto z$, or more formally, it may be written:

$$L = R \qquad (11)$$

in which, R simply indicates the constancy of the ratio, $\frac{L}{l}$ (which is represented by L) of any pair of homologous linear dimensions, as $\frac{A}{a}$ for example. The value of the scale ratio, R, is perfectly arbitrary until the two systems are chosen; but Equation (11) shows, further, that one does not destroy the validity of any correct equation in one of the geometrically similar systems by applying it to any of the others, independent of the value of R. That is what Stokes says so far, and it is evidently quite correct; and with general application to all manner of quantities which have a characteristic constant ratio between their homologous dimensions (whether linear, or otherwise) in the two systems.

Suppose it is now required that there shall be similar motions, in addition to the similarity of forms as required by Equation (11). (This equation is supposed to be applied to all homologous linear dimensions, thus representing an infinite number of equations, but it will be referred to as one equation hereafter.) This is specified by adding Newton's general theorem of similarity, $g \propto \frac{V^2}{D}$ (see Equation (1)). These necessary and sufficient conditions for simultaneous geometrical and dynamic similarity require that Equation (11) shall be satisfied and that,

$$\left(\frac{V^2}{L} = \frac{L}{T^2}\right) = \frac{G}{g} \qquad (12)$$

in which, the notation used on the left relates, respectively, to the ratios of homologues, and of homologous "dimensions" of the homologues, in accord-

ance with the notation on the left of Equation (11). For example, either of the terms, $\frac{V^2}{L}$, and, $\frac{L}{T^2}$, indicates that the ratios of homologous accelerations are equal to certain functions of the ratios of the simpler dimensions, V, L, and T.

Equations (11) and (12) state simply, that Newton's theorem of similitude puts no restriction upon the scale ratio of two similar systems, nor upon the respective values of the homologous accelerations in the two systems, whether such accelerations arise from "extraneous forces" (gravitation, say), or from forces within the system, or from both combined. Relative to these equations, furthermore, Newton's theorem states that, if the experimenter takes care to establish similar motions initially, the similarity will continue. This is entirely independent of the presence or absence of viscous fluids. If Newton's law is satisfied, the systems must be similar, whether viscous fluids are present, or otherwise.

Observe that the extraneous forces may be eliminated, or annihilated, but that there must be left a ratio of homologous accelerations, just as there must be a constant ratio between any two homologues of any other type in similar systems. Stokes makes this necessary assumption, by providing for it in the form, $\frac{c}{T^2}$, and also in his equation,

$$p = \left(\frac{\rho\, u\, x}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

all of which arise from terms which do not contain the viscosity. In order to make the situation perfectly clear, Equation (13) may be expressed in the form:

$$\frac{p}{x} = \frac{\rho\, u}{c}\left(\frac{c}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

or, substituting for $\frac{c}{t}$, its equivalent, u, and transposing:

$$\frac{p}{\rho\, x} = \frac{u^2}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

In other words the ratio, force to mass, varies as $\frac{u^2}{c}$.

Equation (15) is nothing more or less than Newton's law of similarity, $g \propto \frac{V^2}{D}$, or Equation (12), since Stokes intends it to be applied to homologues, and since the ratio of force to mass (left-hand member of Equation (15)) has the dimension, acceleration.

Equation (11) and Equation (12) involve the ratios, L and T, in terms of the two quantities, R and $\frac{G}{g}$, the respective scale ratio and accleration ratio of the two systems. Clearly, these ratios may be chosen arbitrarily without interfering with the consistency of the equations, since the first equation contains only L and does not contain T. In other words, geometric and dynamic

similarity may exist regardless of the particular values of the scale ratio and acceleration ratio in the two systems.

Newton went farther than this in demonstrating his theorem. In the briefest possible language he says (see the theorem as quoted previously) that, if the two equations (Equations (11) and (12)) hold, and the mass ratio is properly adjusted, initial geometrical and dynamical similarity will continue as a necessary consequence.

It is important to emphasize that nothing less than this will do, for the similarity must exist initially (as expressly required by the theorem) so that Equation (11) and Equation (12) are not only sufficient, but necessary; that is, they must hold throughout the entire time and throughout the systems.

For example, if the systems are similar initially, as in the case of Stokes' two pendulums at the points of maximum displacement, the subsequent motions of surrounding frictionless fluids will be similar; or, the motions of surrounding viscous fluids will be similar, if the viscous forces can be adjusted so as to satisfy Newton's theorem, or law of similarity, or, more formally, Equation (11) and Equation (12). One cannot disregard Newton's absolute, necessary, and sufficient condition to sustain dynamical and geometrical similarity.

The only way to explain Stokes' silence on this point is to assume that he really did not mean just what he said, being more immediately concerned with his pendulum analysis than with similarity. However, there can be no doubt that he made the oversight and that it has caused many to repeat his remark. Others, less fortunately, based formulas and calculations of a practical nature upon a misconception. Such oversights are frequently made by writers of unquestionable authority. For example, Rankine has stated* that the coefficient of elasticity of a gas has the form:

$$A = \frac{d\,p}{d\,w} = \frac{\gamma\,p}{w}$$

The slightest consideration shows that this cannot be, since A and p have like dimensions (those of pressure), γ is an abstract number, and w the specific weight of the fluid (Rankine calls it "density"). The nature of the correction is obvious.

This error has been running for the last seventy-five years without correction, apparently without notice, but, thanks to analysis of what is said, rather than reliance upon authoritative literature, the error is self-disclosing by inspection. No better textbook was ever written and none more carefully edited.

Proceeding further, take Stokes' limitation on ratios of homologues, Equation (10), as the additional (not the "sole") condition that systems containing viscous fluids shall be both dynamically and geometrically similar. Then, the three conditions may be represented by Equations (10), (11), and (12). This gives three equations between two unknown ratios, L and T; but only one of the unknowns appears in Equation (11).

* Rankine's "Applied Mechanics" (any edition including that current in 1930), p. 563. See, also, the footnote in *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2209; also, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 836, for another such error by another author.

Equations (10) and (12) present two conditions in terms of two unknowns, L and T. These two ratios, consequently, become determined; so that the condition defined by Equation (11), is no longer arbitrary. Thus, the three ratios, R, $\frac{N}{\nu}$ and $\frac{G}{g}$, are not entirely independent, but any one depends upon the other two.

Solving Equations (10), (11), and (12):

$$L^{\frac{3}{2}} = R^{\frac{3}{2}} = \frac{\frac{N}{\nu}}{\left(\frac{G}{g}\right)^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)*$$

and,

$$T = \frac{\left(\frac{N}{\nu}\right)^{\frac{1}{3}}}{\left(\frac{G}{g}\right)^{\frac{2}{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The conditions of Equations (10), (11), and (12) (all of which are present in Stokes' paper) determine the necessary scale-ratio and time-ratio (Equations (16) and (17)), between the two similar systems in terms of the physical properties of the fluids and the physical conditions existing at the places of the two systems. There can be no dynamically and geometrically similar systems (model and prototype) without all these restrictions, when viscous fluids are present.

This is probably what Stokes intended to read from his paper, but he accidentally overlooked the fact that Newton's theorem must be applied, whether or not the extraneous forces (gravitation, say) are eliminated.

The practical difficulties which arise in constructing models are apparent from Equations (16) and (17). If it is attempted to use the same fluids in model and prototype, the ratio of the coefficients of viscosity is then unity, and it is found that:

$$L = \left(\frac{G}{g}\right)^{-\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In this case it is theoretically wrong to test model and prototype at the same place on the earth's surface, for then L = unity, and the model should be of the same size as the prototype, which would rarely be useful. If the same fluid is present in the two systems, but at different temperatures, such as to change the coefficient of viscosity, then the two conditions are equivalent to two different fluids, and should be so regarded.

On the other hand, if instead of L = unity, $\left(\frac{G}{g}\right)$ = unity (testing at the

* These results were obtained by the writer in studies leading up to his paper entitled "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138. The results of an application to large ice diversion works on the St. Lawrence River, and to the construction of other features, were also described by the writer under the title, "Ice Diversion for St. Lawrence River Power Co.," *Canadian Engineer*, November 25, 1920. See, also, *Engineering Record*, September 25, 1915, p. 377, and *Canadian Engineer*, February 14, 1918, p. 139.

same place on the earth's surface, say), then,

$$L^{\frac{3}{2}} = \left(\frac{N}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which, N and ν are, respectively, the kinematic coefficients of viscosity of the two fluids in prototype and model. Therefore, the two fluids must be different or, if they are the same fluid, then it must be at different temperatures in the model and prototype. Of course, one may have a model of the same size as the prototype with the same fluid at the same temperature in the model and prototype; but that again might not be very useful.

Thus, in this case, the conclusion is reached that, to have the model as small as possible, the ratio, $\frac{N}{\nu}$, should be as large as possible, and the ratio, $\left(\frac{G}{g}\right)$, as small as possible.

Examining Equation (19) more closely, the rule to secure a small-sized model evidently is to select a fluid having as small a kinematic coefficient of viscosity as possible, and also to test the model where the value of g is as large as possible (see Equations (16) and (17)).

Air would not likely make a small-sized model because, owing to the very small value of its density, which occurs in the denominator of the kinematical coefficient, the value of the coefficient is relatively large. Water would serve better, and mercury still better, owing mainly to their increasing densities. Hence, one reaches the astonishing conclusion that an airplane propeller might be tested in a small-sized model in mercury.*

As another example, consider a ship and its model. At corresponding speeds—whether greater or less than critical velocities—the total resistances to a ship and its true model of proper scale in true "model" fluid (say, mercury) would vary exactly as the respective densities, wet areas, and squares of corresponding velocities. This relation between ship and model would hold for all velocities of the ship and model at corresponding speeds (constant-velocity ratio). It would no longer be necessary to treat the skin friction and wave-making resistances separately. This truth illustrates perfectly the real advantage of a true model, namely, that it reduces the entire problem to a comparison of simple ratios, whereas, in general, for arbitrary sizes and shapes, the ratios are not simple. If they were, there would be no need for models.

Summary of Equations of Similitude

Properties of Materials.—Thus far, nothing has been said of the force ratio, or mass ratio. The truth is that both have been determined by the method of making the problem kinematical, which is another process of elimination that has obscured the true relations and added to the confusion previously discussed.

* As suggested in the paper on "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), footnote, p. 1187. It is shown that the model would be one-twenty-fifth full size and would run at one-fifth full linear speed.

As is well known, ordinary mechanics involves only three "fundamental" units, in terms of which all quantities of that science may be measured. At least three equations will be available connecting ratios of homologous quantities in similar systems with the ratios of homologous measures in each of the three fundamental units, L, F, and T (any three units may be taken as fundamental, but here, length, force, and time are selected). Procedure in sciences of more than three fundamental units is the same, there being simply an additional equation for each such unit beyond three.

Summary.—Hence, summarizing the equations of similitude, the four necessary conditions for dynamic similarity among ordinary mechanical systems are:

$$L = R \quad \ldots\ldots (11)$$

$$\left(\frac{V^2}{L} = \frac{L}{T^2}\right) = \frac{G}{g} \quad \ldots\ldots (12)$$

(first property equation),

$$f_1 (L, F, T) = \phi' (P') \quad \ldots\ldots (20)$$

(second property equation),

$$f (L, F, T) = \phi (P) \quad \ldots\ldots (21)$$

To illustrate the use of the property equations it may be explained that, if P' in Equation (20) is to represent the density, this equation would reduce to the form:

$$\frac{F T^2}{L^4} = \rho$$

since the left-hand member expresses the dimensions of ρ. In Equation (21), P denotes the ratio of pertinent homologous properties, such as elastic moduli, electrical constants, etc. If more than one property, or material, must satisfy Equation (21) separately, suitable bodies must be found, made, or arranged, so to do, before similarity can be sustained. If a property (density, or load, say) satisfies the third equation (Equation (20)), then the fourth equation (Equation (21)) is free, as to that property, and may be used with some other property or properties. Thus, P may denote the ratio of homologous viscous, or elastic, pressures, or both, if they should have the same ratio, either by chance or by artifice.

Thus, the ratios L, F, and T, can each be found in terms of the conditions and properties of systems and materials by solving the equations. Conditions and properties include: Terrestrial gravity, internal accelerations, density, viscosity, elasticity, capillarity, mass attraction, electrical forces, etc. Therefore, solving Equations (11), (12), (20), and (21):

$$L = R = \phi_1 (G, \rho, \text{and } P) \quad \ldots\ldots (22)$$

$$F = \phi_2 (G, \rho, \text{and } P) \quad \ldots\ldots (23)$$

$$T = \phi_3 (G, \rho, \text{and } P) \quad \ldots\ldots (24)$$

From these equations, the ratios of homologues of any kind can be found, since any such ratio can be "derived" from its dimensional equation. The specifications for the model may be written from this analysis. Then, if the

model is caused to move with motions similar to those of the prototype, the similarity will continue, because all conditions of Newton's theorem are satisfied.

Example 1.—Write the conditions for dynamical similarity in a model elastic structure. Besides Equations (11), (12), and (20) (see illustration given after Equation (20)), Equation (21) takes the form,

$$\frac{F}{L^2} = E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which, E is the ratio of homologous elastic moduli. If shear as well as extension is taken into account, both E's must satisfy Equation (25). Engineering will be required. True models must be built to specification. They are not more likely to be picked up by chance than is a well-designed structure. If factors of safety enter, still more moduli must satisfy Equation (25). It is as great a mistake to use the same materials, or fluids, in a model and prototype, as it would be to use the same scale. Therefore, solving these simultaneous equations, and assembling relations,

$$L = R = \frac{E}{\rho\, G}$$

or,

$$\frac{R\, \rho\, G}{E} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

$$F = \frac{E^3}{\rho^2\, G^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and,

$$T = \frac{1}{G} \sqrt{\frac{E}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Then, also,

$$L = T \sqrt{\frac{E}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

$$F = E\, L^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

and,

$$V = \frac{L}{T} = \sqrt{\frac{E}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

These equations conform exactly to the dimensions of factors of safety given in the paper on "Ice Diversion, Hydraulic Models, and Hydraulic Similarity" (page 1188), previously mentioned. Equation (26) may be called the "characteristic."

Equations (26) to (31) include the velocities of sound in the solids as well as the velocities of oscillations. In Equation (31), V denotes the ratio of maximum velocities (components) due to vibrations alone; but it is the correct velocity ratio.

These results give great generality for elastic models of suitably selected, made, or arranged materials; but, if model and prototype are of the same materials, the model not only becomes non-representative (elastic properties

change in small sizes of given material), but of limited utility, as may be seen by making $E = 1$ and $\rho = 1$, giving:

$$L = R = \frac{1}{G} \quad \ldots\ldots (32)$$

$$F = \frac{1}{G^2} \quad \ldots\ldots (33)$$

$$T = \frac{1}{G} \quad \ldots\ldots (34)$$

$$L = T. \quad \ldots\ldots (35)$$

$$F = L^2 \quad \ldots\ldots (36)$$

$$V = 1 \quad \ldots\ldots (37)$$

or,

$$V = v \quad \ldots\ldots (38)$$

This latter method ($E = 1$, $\rho = 1$) is to elastic models what "Reynolds' Model Law" is to fluid models; both methods lead to undesirable conditions for model tests. Notice in Equation (26) that $RG = 1$ is the characteristic, since $E = 1$ and $\rho = 1$ (see, also, Equation (32)).

Example 2.—Construct two similar systems, each combining mechanical, elastic, and viscous actions. Suspend a weight by a spiral spring, also making a similarly suspended weight to a smaller scale, such that all manner of oscillations and motions of solids and fluids will be similar. Equation (21) must satisfy two properties (forms two equations):

$$F = k\,L = \mu \frac{L^2}{T} \quad \ldots\ldots (39)$$

Letters refer to ratios of homologues, unless considering one system separately, when they refer to indicated measures. The elastic spring constant and density are, respectively, k and ρ, the former being of the dimensions, pounds per foot. Then, since $R = L$, solve Equations (11), (12), (20) and (21), as follows:

$$\frac{k^3 \rho}{G\,\mu^4} = 1 \quad \ldots\ldots (40)$$

$$L = \frac{k^5 \rho}{\mu^6\, G^2} = \left(\frac{k}{\rho\, G}\right)^{\frac{1}{2}} = \frac{\nu^{\frac{2}{3}}}{G^{\frac{1}{3}}}, \text{ etc} \quad \ldots\ldots (41)$$

$$F = \frac{k^6 \rho}{\mu^6\, G^2} = \frac{k^3}{\mu^2\, G} = \frac{\mu^2}{\rho}, \text{ etc} \quad \ldots\ldots (42)$$

$$T = \frac{\mu^7\, G}{\rho^2\, k^5} = \frac{k}{\mu\, G} = \frac{\nu^{\frac{1}{3}}}{G^{\frac{2}{3}}}, \text{ etc} \quad \ldots\ldots (43)$$

There are four equations involving the three quantities, L, F, and T. The solution demands that these equations be consistent, that is, that they apply simultaneously. The condition is found by eliminating L, F, and T, which gives Equation (40), the "characteristic." Had there been five equations instead of four, there would have been two characteristics instead of one. In fact, a superabundance of properties may introduce any number of characteristics according to the circumstances. It will be seen that some of the quantities involved are arbitrary, as the mass, or weight, of spring, or model;

and this is the case with the elastic constant, which may be assumed at 10 lb. per ft. for the model; or, after testing the full-sized spring, the model must be made to have a constant conforming to the ratio, k. The characteristic (Equation (40)) is equivalent to,

$$\frac{k_1^3\ \rho_1}{G_1\ \mu_1^4} = \frac{k_2^3\ \rho_2}{G_2\ \mu_2^4}$$

each member being dimensionless, and it contains all the properties involved—G, ρ, μ, and k. It corresponds to characteristics in other homologies, Reynolds' number, Froude's number, etc.

Adopting air as the fluid, the following data, in centimeter-gramme-second units, may be found in physical tables, such as International and Critical Tables, and those by Landolt and Börnstein, for the large scale and small scale, respectively. The values used in the problem are merely approximations:

Air temperatures, in degrees centigrade = 260 and 20

$$10^7\ \mu = 2\ 800 \text{ and } 1\ 830$$
$$10^7\ \rho = 6\ 000 \text{ and } 12\ 000$$
$$\nu = 0.467 \text{ and } 0.1526$$

Then, by slide-rule,

$$\frac{M}{\mu} = 1.53 \pm\ ; \frac{\Delta}{\rho} = \frac{6}{12} = \frac{1}{2} \pm\ ; \text{ and } \frac{N}{\nu} = 3.06 \pm$$

In this case, the density for air determines the density ratio for the homology, as all such ratios must be the same throughout for homologues. It will not be difficult to make springs and weights to satisfy this requirement, if the maker can grasp the idea that he is not making springs, but models. The centimeter-gramme-second ratios are the same as the English. The springs are to oscillate under the action of gravity, as well as by reason of their elasticity. Therefore, all homologous accelerations are equal, although they need not necessarily be the same, numerically, as gravity. Thus, the ratio, $G = 1$, and the characteristic gives:

$$k^3 = \frac{\mu^4}{\rho} = 10.96\Big]_{G=1}\ ;\ k = 2.22;\ R = L = \left(\frac{k}{\rho}\right)^{\frac{1}{2}} = \left(\frac{k}{\mu}\right)^2 = 2.104\Big]_{G=1}$$

and, since $G = 1$, the mass ratio is,

$$M = \frac{T^2\ F}{L} = \rho\ L^3 = 4.675\Big]_{\rho=\frac{1}{2}} = F$$

Now, make, calibrate, and adjust, the similar springs. The larger may be of spring steel and the other a model, such that its length, effective density, and elastic constant will be in the proper proportions, L, ρ, and k. The weight (arbitrary) on the smaller spring is 10 lb.; then, on the larger spring, it must be 46.75 lb., the respective masses being $\frac{10}{32.2}$ and $\frac{46.75}{32.2}$ slugs. Thus, to check the calculations, 1.5 ft. being an extension (to one side) of the larger spring,

$$G_1 = \frac{k\ L}{M_1} = 22.9 \text{ ft. per sec.}^2$$

and,

$$G_2 = \frac{10 \times 0.731}{M_2} = 22.9 \text{ ft. per sec.}^2$$

The final check is by Equation (16) thus:

$$\frac{L^3\ G}{N^2} = \frac{9.35 \pm \times 1}{9.36 \pm} = 1$$

which checks.

Of course, Reynolds' equation will now be satisfied, thus:

$$\frac{V\ L}{N} = \frac{v\ l}{\nu} = 18.9$$

This checks exactly, although it is not expressed entirely in English units. It is evident, however, that these results could not possibly have been attained, had Reynolds' equation been considered the sole requirement in addition to geometrical and elastic similarity, as, in that case, Newton's theorem would have been ignored, whereas, it must always be effective in every case of dynamic similarity.

If, now, the springs are extended in proper ratio, R, at any angle to the vertical, and if they are released simultaneously, in their properly tempered atmospheres, the motions will be similar, although compounded of various types of oscillation. By properly proportioned side impulses on the sides of the springs, flexural vibrations may be introduced, as well as longitudinal and pendular oscillations, without destroying the similarity, provided shearing has been introduced into the equations, as mentioned. The springs must be confined in similar spaces.

To attain this result means, of course, that the material, or form (without destroying theoretical similarity, both external and internal), of the model spring must be devised so as to develop an effective shearing modulus of elasticity, or, if necessary, an artificial "modulus", such as will satisfy Equation (21), simultaneously with k. An illustration is given by Equation (39), in which, k and μ are connected.

The resulting motions of the two springs are not synchronous (time scales being different), and would gradually subside unless maintained by some device (clockwork). It should be noticed, however, that if time is reckoned according to the model time-scale (as by a "model" watch), the motions would be synchronous in the sense of similarity. The elastic motions need not be, and probably are not, isochronous.

The problem has been solved by postulating that the air will act as an incompressible fluid. This is accurate for most motions of solids in air, such, for example, as the motions of ordinary machinery, automobiles, and airplanes. If fluid elasticity is to be involved (sound effects, cavitation, etc.), the ratios of corresponding elastic moduli must be made to satisfy Equation (21), just as E does in Equation (25) which is merely a form of Equation (21). In this way, sound, cavitation, etc., should be reproduced in the model.

It is worthy of notice that, whether the cubic strains are isothermal, adiabatic, or whether they vary according to an arbitrary law, the ratio of homologous moduli is the same as the ratio of the homologous atmospheric pressures. This assumes that the ratio of the ratios of the specific heats is equal to unity. In the case of gases, exhibiting a variance in this respect, the ratio of the ratios of the specific heats must be introduced into formulas.

By "specific heat" is here intended the general specific heat, isothermal and adiabatic conditions being merely special cases.

Whether it will be permissible to modify the natural atmospheric pressure, or introduce an artificial "atmosphere", in a model, depends upon conditions in the prototype. If the normal pressure in the prototype nowhere falls below p lb. per sq. ft., then the atmospheric pressure in it may be reduced by p lb. per sq. ft. without otherwise changing materially the dynamic action of the prototype. In such cases the model atmosphere may be adjusted to correspond to such hypothetically reduced atmosphere on the prototype, without making any actual change of the atmospheric conditions in the prototype.

Under these circumstances, there would be a point, or points, in the model at which the pressure would be nil. This last condition determines the least possible pressure of the model atmosphere, for there is no such thing as a negative pressure. It is only a difference in pressure, or its direction, which may be considered as being negative* Such difficulties are not problems of theory. Among many problems, they are merely some which must be solved in the field by the Newtonian model engineer, a type of engineer yet to be developed, of whom much may be expected. A problem involving atmospheric pressures is treated in Appendix I.

Herein lie possibilities for creating a new department of engineering design, that of "model engineering", concerned with the technical methods of constructing models which will perform with theoretically exact similarity. In this way, the most difficult problems of mechanics may be solved by making and shaping materials (including fluids) so as to have desired properties and qualities. This care is especially necessary in designing important structures when the mathematical difficulties render solutions impracticable. This does not necessarily mean difficult constructive problems in making the models, since, in many cases, properly constituted materials and fluids are at hand, or may be made easily.

Many ordinary kinds of forces are properly constituted so as not to interfere with theoretically exact model operation.† Among these may be mentioned all those similar forces which are proportional to homologous areas and to the squares of homologous velocities, such as: (1) Pressures; (2) ordinary skin friction under the V^2 law; (3) reactions of moving masses; (4) "adjusted viscous forces"; (5) static and dynamic effects due to heads of water and of water falling over a spillway; (6) effects of wind on running ice; (7) transportation of detritus and carriage of floating ice; (8) time on a "model" watch; and (9) measurements in "model" units. Time given by the model watch will agree with the actual time. Weighings and measurements by the model weights and measures agree with actual weights and measures for the prototype. Even factors of safety for structures may be included, if models are properly constructed and properly tested. Moreover, if materials are found, made, or arranged to satisfy Newton's theorem of similarity, as many as may be so provided can be used in the model.

* See Lamb's "Hydrodynamics," Third Edition, p. 18, where such pressure changes are mentioned, but without notice of the limitation.

† *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1171.

The effect of varying values of g may be obtained by rotating the model about a vertical axis, if care is taken to place the model in a proper position relative to that axis. This has the effect of increasing g and may be desirable when the properties of the model leave the size to be determined by the acceleration ratio of the homology. Theoretically, when above the surface of the earth, the model should be nearer the earth's center than the prototype, supposing viscous forces to be present. When below the earth's surface, the reverse would be the case. Other conditions than viscosity also affect the size of the model, if similarity is to be obtained. (For effect of pressure, see Appendix I.)

It may be desirable to know the significance of "Reynolds' number." That dimensionless number occurs in many formulas for the flow of viscous fluids;* its form is given in Equation (9). It occurs also in Equation (10). It occurs implicitly in Equations (16) and (17). If L is calculated from Equation (16), then, upon forming $\frac{L\,V}{\frac{N}{\nu}}$, the latter will be found equal to unity. That will mean that Reynolds' number is constant as between similar systems, or, $\frac{V\,L}{N} = \frac{v\,l}{\nu}$ = constant. This may also be calculated from Equation (10) by simple division, having regard to the dimensions of velocity. Therefore, if similarity exists throughout the systems, Reynolds' number will be constant throughout.

It cannot be inferred from this, however, that the systems will be similar if Reynolds' number is constant. In order that this latter state of affairs may hold, the scale of the model relative to the scale of the prototype must be determined so that it will satisfy the three equations, (10), (11), and (12), which is the real reason why Reynolds' number is constant in Equations (16) and (17).†

Example 3.—Let it be supposed that a tank furnishes water to a 5-in. horizontal discharge under 25-ft. head, and that it is desired to make a model of the plant one-fifth of the full size to be operated with water. By Newton's theorem,

$$G = \frac{V^2}{D} = \frac{V^2\,D^2}{D^3} = \frac{1}{D^3} \qquad (44)$$

since Reynolds' number is to be taken constant, and because of the use of water in the model. The velocities are: $V = 8.02\sqrt{25} = 40$ ft. per sec.; and $v = 5 \times 40 = 200$ ft. per sec., since $VD = vd$. Thus, the effective test head is $\frac{200^2}{64.4} = 625$ ft. This head is altogether out of proportion. Reynolds'

* *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, pp. 2212 to 2214, Equations (8) to (13), inclusive. The friction factor, F, in Equation (12), for example, has been shown by experiment to be a function of Reynolds' number; "Similarity of Motion in Relation to the Surface Friction of Fluids," by Stanton and Pannell, Collected Researches, British National Physical Laboratories.

† *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1187, first two paragraphs relative to Equation (20), where this important admonition is given by the writer after careful theoretical studies followed by several years of experimental investigation.

number does not here prove to be an infallible criterion. However, if the horizontal model discharge, alone, is supplied with water from a main under a head of 625 ft., it is supposed that the flows in the discharges will be similar, although the systems are not similar throughout. Thus, $\frac{G}{g} = \frac{200^2}{1} \div \frac{40^2}{5} = 125$, or the value of the acceleration in the model discharge is 125 times normal g, by Newton's theorem.

Now, try a model with mercury at 80° cent., the water in the tank being at 15° cent. The values of the kinematical coefficients are, respectively, $\nu_m = 0.00097$, $\nu_w = 0.0114$, (centimeter-gramme-second); giving $\frac{N}{\nu} = 11.76$. Then, by Equations (16) and (17), $R = 11.76^{\frac{2}{3}} = 5.18$, so that a scale ratio of 1 : 5.18 is correct for the mercury model. Hence, $h = \frac{25}{5.18} = 4.82$ ft.; and $d = \frac{5}{5.18} = 0.965$ in.; the respective head and diameter of discharge for the model plant, which is a perfect model in all respects. The velocity ratio is $1 : \sqrt{5}$, or the velocity in the model discharge is less than one-half that in the larger discharge. No calculations have been made for friction, because the mercury model will reproduce it perfectly; nor does the friction head need to be proportional to the square of the velocity in creating the friction heads in the separate systems, because the similarity will exist between the systems at all corresponding velocities, including pure viscous flow. However, the friction heads, as between model and prototype, will vary as the squares of homologous velocities, also including pure viscous flow.

There are other ways by which to satisfy Newton's theorem, and to adapt the apparatus to viscous fluids. One may assume that the scale ratio and acceleration ratio are known; and then adapt the viscosity ratio to suit the requirements, if suitable fluids can be found, or made. If limited to certain fluids, as in the first part of Example 3, one may let the acceleration ratio be determined from Equations (16) and (17) by introducing the properties of the fluids and the chosen scale ratio, R. In every case, it is necessary to satisfy Equations (16) and (17), and to see that conditions conform to Newton's theorem, in order to have similarity. It is the adjustment of values to satisfy Newton's theorem that suffices to establish a model, and not solely the constancy of Reynolds' (Stokes') number.

In the last case mentioned, (first part of Example 3) when the acceleration ratio is determined in terms of R and N, it may be necessary to use two given fluids, or one and the same fluid, in model and prototype. In either of these events, the force ratio is,

$$F = \rho \nu^2 = \frac{\mu^2}{\rho} \qquad (45)$$

which, of course, is intended to refer to homologues throughout. This force ratio may be found by the general process outlined in connection with and by Equations (20) and (21), as exemplified by Equations (25) to (31) inclusive;

or, that which is the same thing, by solving Equation (20), and the equation formed by the first and third members of Equation (39), for F. In the latter case the result is reached quickly by dividing μ^2 by ρ.

When the two fluids become one and the same fluid, the density ratio and viscosity ratio each become unity, thus also reducing the force ratio (Equation (45)), to unity. This means that the actual total operating force on the model must be the same as the total force operating upon the full-sized prototype. This is an awkward and unnecessary situation, for it frustrates the entire object of a model, in which it is desired to reduce the measures to moderate values. This difficulty has been commented upon by other writers,* and it is a little surprising that none of them has pronounced it an absurd extreme, for the purpose of designing by models. The parallel with elastic models was mentioned in connection with Equations (32) to (38), inclusive, in which the velocity in the elastic model was the same as in the full-sized structure.

Furthermore, the ideas have long been current in scientific and engineering literature that the method of allowing the acceleration ratio to adjust itself to the velocity ratio, is equivalent to "eliminating gravity"; and that the motion of the fluid is "non-accelerated," as in the case of steady flow along a pipe. It is true that the mean velocity of steady flow along a cylindrical pipe is uniform, but the fundamental idea of similarity has nothing directly to do with the nature of the flow in any separate system. The idea concerns itself only with a comparison of homologous conditions in two, or more, distinct systems. Thus, homologous velocities must be in fixed ratios, whether those velocities are uniform or variable, as to time or place, in any given system. The similarity applies to the motion of fluid particles, such that every eddy or irregularity of flow is to be repeated in the model. If this were not true, there would be no such thing as comparing two systems, for it is this elemental similarity which causes like actions. All these eddies and irregularities of flow are the vital agencies which produce the principal friction losses—the main resistances with which it is usually necessary to deal. Consequently, instead of being non-accelerated, the motion is highly accelerated, both longitudinally and transversely, to say nothing of the initial acceleration to introduce the flow, and to maintain it at joints, fittings, and bends. It seems to the writer that the theory of probability might be applied to the flow of water in a manner somewhat similar to that used in the kinetic theory of gases. The study of the nature of such flow (turbulence included) offers one of the great fields for hydraulic and hydrodynamic research.

The use of the same materials in model and prototype leads to certain limiting cases of models. Weber has tabulated† five such cases which compare the operations of models of gravity, mass attraction, viscosity, elasticity, and capillarity.

The writer prefers not to consider these cases as separate types of similarity, but merely as special forms of more general cases due to the operation

* "Model Experiments and the Forms of Empirical Equations," by Buckingham, *Transactions*, Am. Soc. Mech. Engrs., Vol. XVII (1915), Paragraph 30; see, also, "Dimensional Analysis," by Bridgman, pp. 85 and 86.

† Jahrbuch der Schiffsbautechnischen Gesellschaft, 1919, p. 414.

of models of different types, when made of materials identical with those of the prototype. Mathematically, this amounts merely to assigning particular values to the parameters in the equations of each general type of model. This was illustrated when placing E and ρ, each equal to unity, in Equations (26) to (31), inclusive, to reduce them to Equations (32) to (38), inclusive. The latter condition corresponds exactly with Weber's elastic model, but, as shown, is merely a limiting case of one more general.

Taking this view, it would seem better to compare general types of models, rather than special cases of those types. To make such comparison, it is not necessary to go beyond the use of three dimensions, since all homologous ratios may be derived from any three. It will be sufficient to indicate such a general synopsis by comparing the two types which have been previously treated. (See Table 1.)

TABLE 1.—Comparison of Elastic and Viscous Models.

Ratio.	Symbol.	Elasticity.		Viscosity.	
		Dimensions.	Equations.	Dimensions.	Equations.
Scale (linear)	L	$\frac{E}{\rho G}$	(26)	$\frac{\mu^{2/3}}{\rho^{2/3} G^{1/3}}$	(16), (41)
Force	F	$\frac{E^3}{\rho^2 G^2}$	(27)	$\frac{\mu^2}{\rho}$	(39), (42)
Time	T	$\frac{E^{1/2}}{\rho^{1/2} G}$	(28)	$\frac{\mu^{1/3}}{\rho^{1/3} G^{2/3}}$	(17), (43)
Characteristics: General case	.....	$\frac{R \rho G}{E} = 1$	(26)	$\frac{R^3 \rho^2 G}{\mu^2} = 1$	(16)
Special case: Same materials in model and prototype; values of constant ratios	.....	$R G = 1$; $E = 1$; $\rho = 1$	(32)	$R^3 G = 1$; $\mu = 1$; $\rho = 1$	(16), (18)

In some instances the special cases of characteristics can be associated with the names of celebrated investigators. For example, $R^3 G = 1$ indicates the constancy of Reynolds' (Stokes') number, as shown by linear scale ratio and acceleration ratio, thus,

$$R^3 G = L^3 \frac{L}{T^2} = L^2 \frac{L^2}{T^2} = (L V)^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots (46)$$

which is the same as $V L = 1$, or $V D = 1$; that is, respectively, $V L = v l$, and $V D = v d$, the usual form of statement that Reynolds' number (Stokes' number) is constant for a given fluid. For purposes of definition, it seems better to state such dimensionless numbers in terms of the constants of the particular problem involved; that is, in terms of given data. This has been done in Table 1.

It is instructive to notice that the dimensions of L, F, and T, are given in terms of the "dimensions"—acceleration, density, viscosity, elasticity, etc.—

in which, by making the problems kinematical, the number of arbitrary dimensional ratios may be reduced to two, the least possible number. As an example, the dimension, L, in viscosity may be expressed in the two dimensions, N and G, each being necessary dimensions in a viscous model. Theoretically, there are three dimensions, μ, ρ, and G. In elastic models the three dimensions are, E, ρ, G, etc. In this last case, to secure similarity, E must be the same for all types of moduli involved. Observe that in each case the ratio, E, is that of two similar moduli.

It will be noticed, of course, that the general case of the characteristic for viscous models, in Table 1, is precisely the form expressed by Equations (16) and (17).* Reynolds' number is a special case of this general characteristic of viscous models. It may be worth while to emphasize that this characteristic, and also that Equations (10), (11), and (12), are satisfied by straight-line viscous flow, as well as by turbulent flow, when similarity holds.† There is no essential difference in these two types of similarity; both of them conform exactly to Newtonian, which alone is dynamic similarity. If one will substitute the dimensions of N and ν in Equations (16) and (17), either of the two forms of Newton's theorem may be made to appear. When this criterion is satisfied throughout systems containing viscous fluids (model and prototype), and only then, can similarity exist throughout the systems.

It seems to be somewhat of an open question, whether similarity will establish itself, as in a pipe, but Stanton and Pannell's experiments, previously mentioned, go a long way toward showing that it will. These experiments, properly interpreted, are to hydraulics what Stokes' investigations were to hydrodynamics. It is hoped that experiments of the same nature will be made for differing degrees of roughness, and for differing shapes of channel boundary.‡ Every hydraulic engineer should study Stanton and Pannell's paper on "Similarity of Motion, in Relation to the Surface Friction of Fluids."§ The word "similarity," in the title of this paper, is used in a somewhat different sense from that adopted in the writer's present paper unless the conditions of the test were such as to satisfy Newton's theorem.

Of course, Reynolds' number was satisfied in Stanton and Pannell's experiment, so that theoretically exact dynamic similarity may have been in operation; but it is certain that Newton's theorem must be shown to hold in any case, before an inference to that effect can be sustained.

Relative to the significance of Reynolds' number, the functional relation between the coefficient of hydraulic friction and the value of that number, as exhibited in the experiments of Messrs. Stanton and Pannell, shows that all hydraulic friction, of whatever nature, including turbulence, depends primarily and ultimately upon viscosity.‖ This is not a common view. It is, in fact, one

* See, also, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1186, Equation (19).

† See *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, pp. 2207–2214, for an explanation of hydraulic resistance of all kinds.

‡ "Strömungswiderstand in rauhen Röhren," von K. Fromm. *Zeitschrift für angewandte Mathematik und Mechanik*, Band 3, Heft 5. Gives results of recent experiments on resistances in channels of different degrees of roughness and of different shapes.

§ Collected Researches, National Physical Laboratory (British).

‖ *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2214.

that has been rejected by numerous writers of profound attainment,* but the writer clings to the affirmative for the reasons cited in the former reference.†

Conclusions

It is concluded that the present usage of models is not wholly consistent with sound theory. Newton's theorem for conditions of dynamical similarity has been gradually fading from view, while the fallacious idea that there are differing kinds of similarity has been unduly propagated. Nothing can be farther from the truth than the idea of differing kinds of similarity. All the variously designated "laws" of similarity reduce immediately to Newton's one unique definition and theorem. When that theorem is fully satisfied, Reynolds' number will be satisfied. Even in the absence of fluids of any kind, if Newton's theorem is satisfied under the conditions of acceleration in Equation (44), Reynolds' number will be satisfied ($V D = 1$), but that number may be satisfied without the existence of similarity. Moreover, except as just stated, and in the case mentioned, Newton's theorem may hold and similarity may exist, when Reynolds' number is not satisfied (for example, when, in the absence of viscous fluids, $v d$ is not the same in model and prototype).

Besides the establishment of initial similarity, the necessary and sufficient conditions to establish a model are, that the size of the model, the coefficient (kinematic) of viscosity, and the acceleration ratio, shall all be determined so that Equations (10), (11), (12), and (16)—in the general case Equations (11), (12), (20), and (21)—shall be satisfied. If this is not done, it is not known exactly what errors result, and one must rely upon the supposition that the effects of viscosity (and any other properties) in distorting the model are not great. Perhaps the important conclusion is that a thorough investigation should be made to ascertain what materials and fluids will serve best as materials for models, and to synthesize, if possible, materials which are capable of having their "physical properties" adjusted. By "physical properties" is meant, not only the usual properties of materials, but artificial properties produced in a practical way so as to serve in the manner desired. It is not improbable that, in due time, the stream lines and eddies of flowing water will be produced by mechanisms in the absence of fluids altogether.

There is a great field for the development of a corps of specialists to build models to the specifications indicated by the Newtonian theory, which is the only sound theory for models.

The final paragraph of Appendix I shows that an homology of elastic solids and incompressible fluids is mathematically identical with an homology of elastic fluids and incompressible solids. More generally, any homology of solids, liquids, and gases of varying (constant) characteristics (properties), will remain a homology, in the sense here used, providing the physical properties of the constituents and the other internal and extraneous conditions, instantaneously adjust themselves (or remain constant, as adjusted) to the requirements of Newton's theorem.

* *Transactions*, Am. Soc. M. E. (1915), p. 281, "Model Experiments and the Forms of Empirical Equations," by Buckingham; Encyclopædia Britannica, Eleventh Edition, Vol. 27, p. 736; "Dimensions of Units," by Larmor; "Dimensional Analysis," by Bridgman, p. 85; and "Hydrodynamics," by Lamb, Third Edition, 1906, p. 593.

† *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2214.

APPENDIX I

As further illustration of the nature and use of the dimensional forms for L, F, and T, given in Table 1, and as illustrating the method of treating compressibility of fluids, such as mentioned after solving Example 2, involving Equation (39), it will be interesting to examine the possibility of testing an airplane propeller in air under pressure as has been suggested.* This, of course, would increase the density and, consequently, diminish the kinematic viscosity, so as to admit of an accurate test of a model of reduced scale, assuming that the viscosity is not seriously affected adversely by an increase of pressure. It will then be of additional interest to see if the conditions of test can be improved by substituting a different gas in the model. The method of substituting a liquid (mercury, say) to represent the air, has been mentioned in the remarks following Equation (19). The question of "slip" is discussed in treatises on the kinetic theory of gases, and is also referred to by Lamb.†

The specifications of a gas should always relate to standard conditions of pressure and temperature, usually 1 atmosphere, and 0° cent. Then the gas equation, $P = c\, w\, \theta$, determines the pressure for any other conditions, the standard conditions being indicated by $P_s = c\, w_s\, \theta_s$, assuming that c remains constant. Whence,

$$\frac{P'}{P_s'} = \frac{w'\, \theta'}{w_s'\, \theta_s'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

and,

$$\frac{p_m}{p_s} = \frac{w_m\, \theta_m}{(w_s)\, \theta_s} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

in which, the primed symbols may relate to the prototype, and the other symbols to the model. The subscript, s, refers to standard conditions of pressure and temperature. The symbols, w, refer to the specific weight, or to the density, indiscriminately, as long as the acceleration of gravity has its standard value. In either case, w is the reciprocal of the specific volume of the gas, the volume in terms of which the gas equation is so frequently written. Observe that the model and prototype may contain either the same, or different, gases.

Then, the ratios of homologues, prototype to model, become:

$$P = \frac{P'}{p_m} = \frac{w\, \theta}{w_s} \left| \begin{array}{l} P_s' = p_s \\ \theta_s' = \theta \\ \theta = \dfrac{\theta'}{\theta_m} \\ w = \dfrac{w'}{w_m} \\ w_s = \dfrac{w_s'}{(w_s)_m} \end{array} \right. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

and,

$$\rho = w = P\, \frac{w_s}{\theta} \Big]_{G' = g_m = g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

* See *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), footnote, p. 1187.

† "Hydrodynamics," Third Edition, p. 594.

In Equation (50), the equality of the specific weight ratio, w, to the density ratio, ρ, refers to the comparison of conditions in the prototype and model as affected by standard gravity (equality of g in model and prototype). If the acceleration is really different in the model, then the specific weight ratio, referred to the resulting different acceleration ratio, is not the same as the density ratio. Of course, the density ratio is unaltered. Thus,

$$\bar{w} = \rho\, G = w\, G \quad \ldots\ldots (51)$$

in which $\bar{w}$ indicates the new specific weight ratio. It may be represented by $w\, G$.

If it is assumed, as usual, that the viscosity of the gas is independent of the pressure,* then, for fixed temperatures in model and prototype, the absolute viscosity of each gas will be determined by its temperature. The resulting constant viscosity ratio will be indicated by μ, which, accordingly, becomes a constant of the problem.

Bearing in mind the foregoing relations, and recalling the solution of Equations (11), (12), (20), and (21), (Newton's conditions of similitude), as indicated in Table 1, the following simultaneous equations may be written:

$$R^3 = \frac{\mu^2}{\rho^2\, G} \quad \ldots\ldots (52)$$

$$F = \frac{\mu^2}{\rho} \quad \ldots\ldots (53)$$

$$T^3 = \frac{\mu}{\rho\, G^2} \quad \ldots\ldots (54)$$

$$\rho = \frac{P\, w_s}{\theta} \quad \ldots\ldots (55)$$

$$F = P\, R^2 \quad \ldots\ldots (56)$$

Equation (56) represents a very obvious equality.

As mentioned in the text, any quantity of ordinary mechanics can be expressed in terms of any three "fundamental" dimensions. Let these three dimensions be the most important three constants of the problem, say, R, μ, and $\frac{\theta}{w_s}$; or, P, μ, and $\frac{\theta}{w_s}$, if the pressure is more important than the scale ratio. Hence, in the former case, eliminating ρ, G, and P, the dimensions L, F, and T, are expressed in terms of the former set of dimensions, as follows:

$$L = R \quad \ldots\ldots (11)$$

$$F = \mu\, R \sqrt{\frac{\theta}{w_s}} \quad \ldots\ldots (57)$$

$$T = R \sqrt{\frac{w_s}{\theta}} \quad \ldots\ldots (58)$$

from which the ratio of any pair of homologues may be found. Thus, for example,

$$G = \frac{L}{T^2} = \frac{1}{R} \left(\frac{\theta}{w_s} \right) \quad \ldots\ldots (59)$$

* "Hydrodynamics," by Lamb, p. 536.

$$\rho = \frac{T^2 F}{L^4} = \frac{\mu}{R}\sqrt{\frac{w_s}{\theta}} \quad \text{.....(60)}$$

$$P = \frac{F}{L^2} = \frac{\mu}{R}\sqrt{\frac{\theta}{w_s}} \quad \text{.....(61)}$$

$$V = \frac{L}{T} = \sqrt{\frac{\theta}{w_s}} \quad \text{.....(62)}$$

If it is desired to have a small model, R will be chosen, necessarily, as large as practicable, numerators of ratios relating to the prototype. This would tend to make the ratio, P, smaller, or the pressure in the model would be increased as the size of the model is decreased. This puts a practical limitation on the size of the model. However, by taking θ as large as possible, and w_s as small as possible, that is, keeping the temperature of the model atmosphere low, and selecting a fluid for the model with as great a density (under standard conditions) as practicable, the conditions of operation will be improved. If air is used in both model and prototype, the density of the air in the model may be increased by increasing its pressure. There does not appear to be any great changes in μ for ordinary changes of temperature, but one should use a model gas having as little viscosity as possible. Notice that diminishing θ (in case of most gases) has a like effect upon μ. In following the discussion, the text shows when a symbol relates to a ratio, and when it relates to a quantity in one of the systems only (model or prototype).

The investigator is now in a position to examine any homology which contains either the same, or different, fluids, especially for the purpose of determining the suitability of a particular fluid for use as a "model" of another fluid. Thus, let it be required to examine air for use as a model of itself, say, for the purpose of testing a model airplane propeller of small size in an atmosphere of modified air.

For this case, $w_s = 1$, since the specific weight of air is always the same for standard conditions; but θ may be modified, say, from 40° cent. (hot summer weather) to 0° cent., in the model; thus $\theta = \frac{313°}{273°} = 1.146$ (slide-rule), while μ, for these conditions, will be about $\frac{190}{170} = 1.118$; therefore, the dimension, $\sqrt{\frac{\theta}{w_s}}$, becomes 1.070. The size of the model may be tried at one-sixteenth the full size of the propeller, $R = 16$. Theoretically, the space in which the model is tested should be indefinitely large, all near-by objects being carefully made to scale, yet, for convenience, the model may be tested in a tunnel, or in a large pipe, provided the model is sufficiently small, relatively thereto. Equations (11), (57), and (58) then give, $L = 16$; $F = 19.14$; and $T = 14.94$, which, substituted in the first forms of Equations (59), (60), (61), and (62), or, if desired, the values of μ, R, and $\frac{\theta}{w_s}$, directly substituted in the second forms of Equa-

tions (59), (60), (61), and (62), give $G = \frac{1}{13.95}$; $\rho = \frac{1}{15.34}$; $P = \frac{1}{13.36}$; and $V = 1.07$.

Thus, the greatest difficulty with an air model is caused by the relatively high pressure in the model (about $13.36 \times 14.7 = 196.5$ lb. per sq. in.); but this would operate nicely if it were practicable to use such pressure.

In order to ascertain whether it is possible to improve the pressure condition in the model, another fluid, different from air, may be examined. An improvement in this respect will be made, if a gas can be found (it has been shown previously that a liquid such as mercury might serve) having a greater standard density than that of air. There are several such gases and vapors, which should be thoroughly investigated.

For present purposes, it will be assumed that a suitable gas can be found of viscosity (absolute) 1.111 times that of air under the actual conditions of the test. (It would be better to have its viscosity less, but it is assumed greater, for illustration). Thus, μ is about 0.9, which will be considered as the value applying for the temperature conditions of the test, since, as mentioned previously, the viscosity of the gas is independent of the pressure. The density ratio under standard conditions will be taken as $\rho = w_s = 0.133 = \frac{1}{7.525}$.

The scale ratio, R, will be 16, as before.

With these three constants (μ, ρ, and G) fixed, the problem becomes determinate by Equations (11) and (57) to (62). Then, by introducing the numerical values of the constants, making the calculations as suggested, and as illustrated in the previous calculations, and by assembling all the formulas and results obtained in this Appendix, Table 2 may be compiled as shown.

A further improvement can be made if μ is reduced in the model. This suggests the idea of devising a method of synthesizing, or treating, a fluid in such manner as to secure a decreased viscosity without altering seriously the value of the density ratio. This makes the molecules of the gas (regarded as a homogeneous fluid) more "slippery," or, perhaps it would be more accurate to say, rendering the molecules less "coherent," so as to admit of rapid shearing distortion with less resistance.

Hence, it appears, that a different gas for the model may admit of reduced pressure in the testing pipe and, also, approximately, in the case examined herein, less than one-half the pressure required in the prototype, in this case, air. Moreover, the velocity in the model, for such change, is reduced to about one-third the velocity in the prototype; it is reduced to about one-third its value for the test of the model propeller in air.

The acceleration ratio, G, is not here due directly to gravity (terrestrial), but relates to homologues in the form of eddies and turbulence generally, as well as to centripetal accelerations at bends, etc. In fact, only the portion of space surrounding (and including) the model propeller, and along the testing pipe, or tunnel, forms the model. Whether the restricted dimensions of the pipe will have a material effect upon the action of the model, is a question which must be considered by the engineer.

TABLE 2.—Conditions for Test of Model Airplane Propellers in Gases Under Pressure (Study of Possibilities).

Symbol.	Dimensions. In Terms of: μ ρ, and G.	Dimensions. In Terms of: μ, R, and $\frac{\theta}{w_s}$.	Dimensions. In Terms of: μ, P, and $\frac{\theta}{w_s}$.	Illustrative Numerical Values. Test of Model in: (a) Same Gas (Air). $w_s = 1$; $\theta = 1.146$; and $\mu = 1.118$.	Illustrative Numerical Values. Test of Model in: (b) Different Gas. $w_s = 0.133$; $\theta = 1.146$; and $\mu = 1.111$.
L.....	$\frac{\mu^{2/3}}{\rho^{2/3} G^{1/3}}$	R	$\frac{\mu}{P}\sqrt{\frac{\theta}{w_s}}$	16	16
F.....	$\frac{\mu^2}{\rho}$	$\mu R\sqrt{\frac{\theta}{w_s}}$	$\frac{\mu^2}{P}\left(\frac{\theta}{w_s}\right)$	19.14	42.3
T.....	$\frac{\mu^{1/3}}{\rho^{1/3} G^{2/3}}$	$R\sqrt{\frac{w_s}{\theta}}$	$\frac{\mu}{P}$	14.94	5.45
G.....	$\frac{L}{T^2}$†	$\frac{1}{R}\left(\frac{\theta}{w_s}\right)$	$\frac{P}{\mu}\sqrt{\frac{\theta}{w_s}}$	$\frac{1}{13.95}$	$\frac{1}{1.856}$
ρ.....	$\frac{T^2 F}{L^4}$†	$\frac{\mu}{R}\sqrt{\frac{w_s}{\theta}}$	$P\frac{w_s}{\theta}$	$\frac{1}{15.34}$	$\frac{1}{52.20}$
P*.....	$\frac{F}{L^2}$†	$\frac{\mu}{R}\sqrt{\frac{\theta}{w_s}}$	P	$\frac{1}{13.36}$	$\frac{1}{6.056}$
V.....	$\frac{L}{T}$†	$\sqrt{\frac{\theta}{w_s}}$	$\sqrt{\frac{\theta}{w_s}}$	1.07	2.94
Gauge pressure in model (14.7 lb. less than absolute pressure) 182.3.....					74.3

* Since PR is a constant in these similar systems, the bursting stress on the sides of pipes (or testing tunnels) of homologous thickness is constant; thus, the pipe may be made safer as the size of the model (including the pipe) is decreased. This favors small models.

† In terms of L, F, and T, may be used to reduce to results of the following columns.

It can easily be shown that Equations (11), (57), and (58) and their various forms as indicated in Table 2, are identical with Equations (40), (41), (42), and (43) in the problem of the elastic pendulums oscillating in gases, or fluids, of an incompressible nature. For example, observing that $k = PL$, in Equations (40), (41), (42), and (43), and that by Table 2, $\rho = \frac{P w_s}{\theta}$ and $G = \left(\frac{P}{\mu}\sqrt{\frac{\theta}{w_s}}\right)$, one may substitute in, say, Equation (41), for k, ρ, and G, so as to reduce that equation to an identity, that is, to $L = L$. A similar process reduces all the other equations, in this group, to identities, thereby showing the equivalence of the algebraic forms in Equations (41) to (43), and (11), (57) and (58); or in Equations (40) to (43) and Table 2.

In the case of Equations (11), (57), and (58), or Table 2, the fluid is assumed to be elastic and the solids, incompressible. In the case of Equations (40) to (43), the solids are elastic and the fluids incompressible. Of course, any, or all, the materials may have the property of elasticity or any other properties, if, as explained in Example 2, these properties are suitably "adjusted."

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FORMULAS FOR RAINFALL INTENSITIES OF LONG DURATION

By Merrill M. Bernard,* M. Am. Soc. C. E.

Synopsis

The object of this paper is to make available for any locality within the limits of the study, rainfall intensity formulas for frequencies of 5, 10, 15, 25, 50, and 100 years, applicable to rainfall duration periods of 120 to 6 000 min.

The need for some means of obtaining for a specific locality, a knowledge of rainfall intensities, durations, and frequencies, is a vital one. This is especially true of those outlying communities which do not have the advantage of long, local rainfall records.

Introduction

The following treatment of rainfall intensities has resulted from the writer's experience in applying the rational method of flood-flow analysis, so successfully utilized in storm-sewer design, to the broader field of small stream flow, particularly agricultural drains. Its scope, therefore, will be limited to that section of the United States lying between the 101st and the 81st meridians, wherein land drainage is a matter of great interest. The writer has found that while some thought† has been given to this problem, much remains to be done by way of engineering research.

The rational expression of flood discharge is:

$$Q = CiA \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which,

Q = discharge, in cubic feet per second.
C = a coefficient expressing the ratio of run-off to rainfall.
i = rainfall intensity, in inches per hour.
A = area of water-shed, in acres.

Note.—Written discussion on this paper will be closed in **February, 1931**, *Proceedings*.

* Cons. Civ. Engr., Crowley, La.

† "Maximum Stream Flow", by C. E. Grunsky, Past-President, Am. Soc. C. E., *Public Roads*, June, 1926; "Run-Off from Small Agricultural Areas", by C. E. Ramser, M. Am. Soc. C. E., *Journal of Agricultural Research*, Vol. 34, No. 9; "Determining Run-Off from Local Rainfall Records," by Merrill M. Bernard, M. Am. Soc. C. E., *Proceedings*, Louisiana Eng. Soc., Vol. LX, No. 5.

Two marked distinctions between the use of Equation (1) in the field of storm sewers and that of agricultural drains are to be noted: First, the run-off coefficient, C, has an entirely different range of values; and, second, concentration periods, and, therefore, duration periods, are much longer.

It is with the rainfall intensities of storm periods longer than 120 min. that this paper deals.

Rainfall Intensity, Duration, and Frequency

With the longer concentration periods encountered in agricultural drainage areas comes the need of a more complete knowledge of rainfall intensities for duration periods longer than 120 min. One, two, three, and four-day storm periods are successions of highly intense downpours, separated by intervals of light, or of no, rainfall. In the case of sewers, each storm phase may be considered as an individual occurrence. Concentration periods are usually less than 2 hours, and the drainage system has the opportunity to clear itself before the effect of the next period of intense rainfall is felt. The higher degree of saturation throughout the remainder of the storm period is the only relating factor.

This condition does not prevail in the larger, more pervious water-sheds. The concentration time, comparable with the inlet time in storm-sewer design, will range from 50 to 1 200 min. on a water-shed of 1 000 acres, depending on the slope, the shape of the area, and the degree of development. Lower velocities are responsible for longer time of flow through the drains themselves, so that, for drainage areas of 300 sq. miles, one is dealing with concentration periods, and, therefore, with rainfall duration periods, of 4 000 min., or more.

The slow movement of run-off over the ground, through furrows and unimproved drains, and the relatively greater effect of retention and retardation, tend to equalize the rate of run-off resulting from a series of downpours, which combined make up the typical storm period of from 1 day to 4 days duration. There is little to dispute the fact that the net effect of a 3-day storm having three to five phases, is the same as that of a hypothetical storm of average intensity, determined by dividing the total rainfall depth, in inches, by the duration of the storm, in hours. Adolph F. Meyer, M. Am. Soc. C. E., has reached this conclusion* regarding rainfall-duration periods of from 30 to 120 min., and the conception may be carried into the longer periods with added certainty.

A knowledge of the frequency with which a rainfall intensity of a given duration will be reached or exceeded in a locality, is as essential to the intelligent design of the conduit which is to remove the run-off resulting from such rainfall, as a knowledge of the degree of the intensity itself. Formulas which classify rainfall intensities as "maximum", "ordinary", and reached or exceeded "occasionally" or "frequently", have no place in the design of a sewer or drain where all other factors are given determinate values.

* "The Probable Frequency of Given Rates of Rainfall", *Engineering News-Record*, Vol. 87, p. 1068.

Frequency is the measure of the efficiency; it explains and justifies the occasions when the structure is taxed beyond the capacity for which it was designed.

Frequency, when applied to meteorological phenomena, does not mean that at regular intervals the phenomenon will occur. Instead, for a record covering a selected period of years, it can be expected that a rainfall of given average intensity and duration will be reached or exceeded, about once in so many years. Dividing the period by the number of such happenings, the quotient obtained is the average length of time, in years, during which the phenomenon has happened once.

This conception of frequency should not detract from its importance. In establishing the limit beyond which the designed capacity will be exceeded, the capitalized loss or damage will be no different at the end of the period analyzed whether a particular event occurred at regular intervals or in two consecutive years.

Intensity Formula Types

The almost exclusive interest in rainfall-duration periods of from 5 to 120 min. has developed the following type of rainfall intensity formula,

$$i = \frac{a}{t + b} \qquad (2)$$

in which,

t = duration of rainfall, in minutes.

a and b = coefficients having definite values for any locality.

The general application of Equation (2) to rainfall intensities for this range of duration periods is demonstrated by Mr. Meyer,* who has made available for the Eastern United States a series of such formula groups.

The results of this and similar studies,† however, show that the relation between rainfall intensity and duration is more accurately expressed by the parabolic formula of the type:

$$i = \frac{C}{t^n} \qquad (3)$$

in which,

C = a coefficient, depending for value on locality.

n = an exponent, depending for value on locality.

A distinction between these types may be drawn by considering Equation (2) as applying to single storm phases lasting less than 2 hours, while Equation (3) is intended to express average intensities of storm periods of longer duration.

One cannot be extended into the field of the other without confusion and error. Fig. 1 compares the Meyer curves, extended, with those of the writer for Columbia, Mo. The curve groups are in agreement between the 60-min.

* "Elements of Hydrology," 1928.

† "Storm Rainfall of Eastern United States", Technical Repts., Miami Conservancy Dist.; "Rainfall and Run-Off Studies," by C. E. Grunsky, Past-President, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 76; "American Sewerage Practice," Vol. 1, Second Edition, p. 260; and "Determination of Storm-Water Run-Off", *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 1182.

and 200-min. intervals, but the former rapidly falls when produced beyond its intended limit of 120 min. It will be noted, from the plotted observational data, that all but three of the points lie above the Meyer 5-year frequency curve, which, considering the length of the record, shows the position of the curve to be in error.

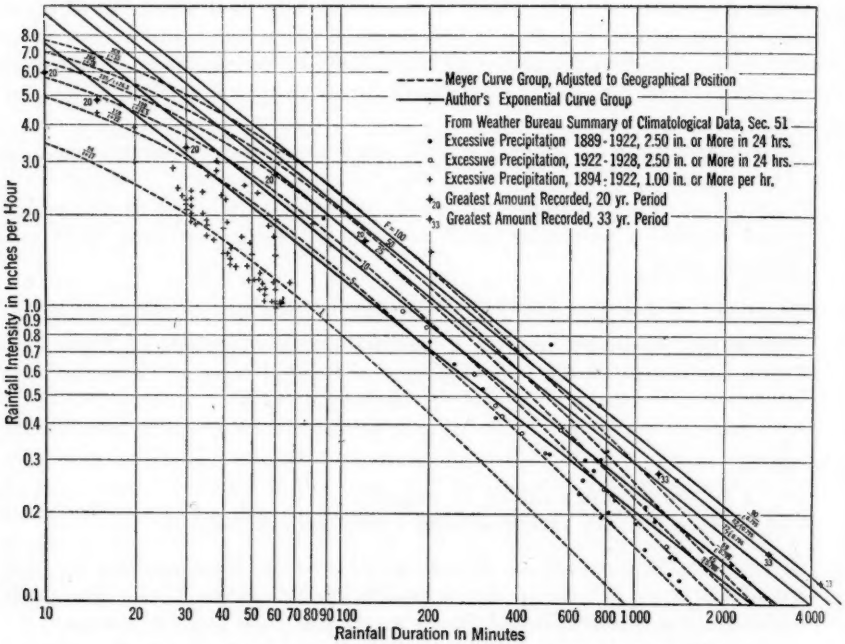

FIG. 1.—COMPARISON OF RAINFALL INTENSITIES FOR COLUMBIA, MO.

It is seen from Fig. 1 that, for duration periods shorter than 60 min., the curves separate. The exponential values increase rapidly, until at 10 min. they are about twice the Meyer values and obviously in error.

C. E. Grunsky, Past-President, Am. Soc. C. E., has made* a comparison of two intensity formulas: The New York formula,

$$I = \frac{150}{t + 20}$$

and the parabolic formula,

$$I = \frac{14.5}{\sqrt{t}}$$

which brings out the limitations of the former type when applied to periods longer than 2 or 3 hours.

The slight advantage involved in the easy determination of a value for t does not warrant the use of 0.5 as its exponent. The proper value ranges from 0.70 to 0.82, as will be shown.

* "Rainfall and Run-Off Studies", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 75-76.

At several of the United States Weather Bureau stations in the area, records of rainfall intensity and duration for storms of 2.50 in., or more, in 24 hours have been kept and are tabulated to 1922. The average length of record is 32 years. Fig. 1 compares the writer's curve group with such observational data at Columbia, Mo. The time interval, 120 to 1 440 min., is well represented and the example given is representative of the eighty quadrangles studied.

Others* have noted the gap in both observational data and rainfall intensity studies which prevails between the 120 and 1 440-min. duration periods, as graphically shown in Fig. 2. This range of rainfall duration applies to many small flood-flow problems, and it also embraces the critical condition where two or more storm phases closely follow each other with little cessation between them.

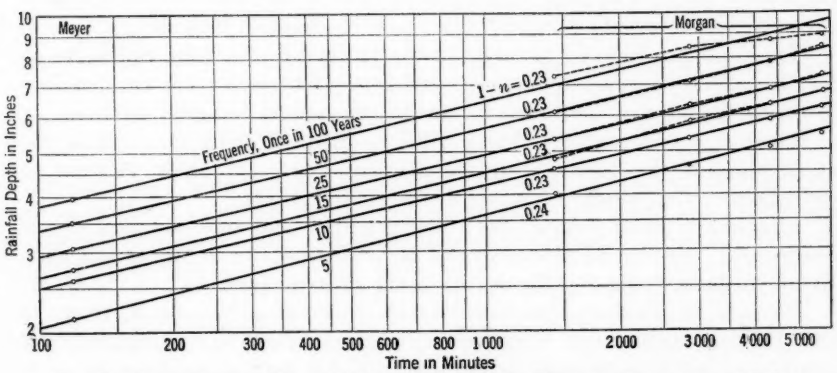

FIG. 2.—ILLUSTRATING GAP IN MEYER AND MORGAN STUDIES, QUADRANGLE 15-*E*.

BASIC DATA

Two studies of the rainfall records of the Eastern United States, outstanding in magnitude, have been made. One is by Mr. Meyer,† who has analyzed approximately 2 000 storms and as a result has presented a series of formulas expressing the relationship between rainfall depth, intensity, and duration, and geographical location. The other is the work of Arthur E. Morgan, M. Am. Soc. C. E., who, in the interest of maximum flood flow, made an exhaustive study and analysis of the rainfall records of the Eastern United States for the Miami Conservancy District.‡

These works are similar in the following respects:

(1) Both cover the eastern half of the United States.
(2) Both are based largely on Weather Bureau data.
(3) Both assume that, within a limited area, the sum of the records of the several stations involved is equivalent to a long-time record of one representative station.

* "Additional Meteorological Data Needed by Engineers", by Robert E. Horton, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 82, p. 614; and "The Frequency of High Rates of Rainfall", by Allen Hazen, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 87, p. 858.

† "Elements of Hydrology", 1928.

‡ "Storm Rainfall of Eastern United States", Technical Repts., Miami Conservancy Dist.

The studies differ in that:

(*a*) Meyer's purpose was the development of intensity formulas for heavy storms of short duration, while Morgan was interested in the longer storm periods of 1 to 6 days.

(*b*) Meyer has presented his work in the form of intensity formulas. Morgan has presented his in a series of isopluvial charts which give the relation of rainfall depth to duration and frequency.

(*c*) The Meyer frequencies are once in 1, 2, 5, 10, 25, 50, and 100 years. The Morgan frequencies are once in 15, 25, 50, and 100 years.

Method of Analysis

In combining the Meyer and Morgan data, the writer has utilized a relationship which has proved to be consistent in eighty quadrangles of the Morgan designation, embodied in his study. The basic data are: (*a*) Meyer's rainfall depths for 120 min., for frequencies of 5, 10, 25, 50, and 100 years; and (*b*) the isopluvial charts, by Morgan,* for 1 to 4-day rainfall, inclusive.

The study has resulted in twelve charts (Figs. 4 to 15), from which, for any locality within the area, values of C and n may be selected, for frequencies of 5, 10, 15, 25, 50, and 100 years, for use in the general formula, Equation (3).

The development of the charts will be explained using Quadrangle 15-*E* as an example, this representing the average as to geographical location and the accuracy with which the curves fit the plotted points.

First.—A printed record sheet was prepared and one assigned to each quadrangle. The record sheet provided for all data shown in Tables 1, 2, and 3. Every quadrangle was rigidly treated in the same manner.

Second.—Isopluvial charts for 120-min. duration periods for 5, 10, 25, 50, and 100-year frequencies were then prepared. This was accomplished by plotting on large scale maps of the area, divided into 2° quadrangles as are the Morgan charts, lines representing the mean of each group.† These isopluvial lines were given the Meyer depth values for 120 min., separate charts being used for each frequency. The pluvial index for each quadrangle, referred to the center, was interpolated and recorded on the charts. This, then, made available isopluvial charts for 120, 1 440, 2 880, 4 320, and 5 760-min. duration periods.

Third.—The pluvial indices for each quadrangle were tabulated. Table 1 gives these data for Quadrangle 15-*E*. The numerical indices of the Morgan charts were recorded, but only the interpolated values were used in plotting the curves. In so doing the writer has taken advantage of the careful adjustment of the isopluvial lines to the weighted length of record of each quadrangle.

Fourth.—The data in Table 1 were plotted logarithmically as shown in Fig. 2. First, frequency was plotted against depth for time, 120 min. (Fig. 3), and the depth value for a frequency of 15 years was obtained by interpolation. Then, for frequencies of 15, 25, 50, and 100 years, time was plotted against depth (Fig. 2), resulting in the points controlling the upper four curves which were placed so as to fit the points best. The 120-min. depth values were allowed to control the lower ends of the curves in each case.

* Technical Repts., Pt. V, Miami Conservancy Dist.

† "Elements of Hydrology," 1928, p. 166.

The difference in the values given by any point and its equivalent curve value did not exceed 10% and the average difference in the eighty quadrangles was 3 or 4 per cent. As a general rule, the percentage of difference varied inversely with the record length of the quadrangle.

TABLE 1.—PLUVIAL INDEX, QUADRANGLE 15-*E*.

(*A* = interpolated values; *B* = numerical value from isopluvial charts.)

Frequency, in years	Time, in Minutes.								
	120	1 440		2 880		4 320		5 760	
		A.	*B.*	*A.*	*B.*	*A.*	*B.*	*A.*	*B.*
5	2.13	....	....	....	....	....	....	....	....
10	2.60	....	....	....	....	....	....	....	....
15	2.75	4.8	5.1	5.8	5.7	6.3	6.0	6.7	6.5
25	3.05	5.3	5.6	6.3	6.2	6.8	6.8	7.3	7.5
50	3.50	6.1	6.3	7.1	7.0	7.8	7.5	8.4	8.7
100	3.97	7.3	7.8	8.4	8.8	8.7	8.9	8.9	9.2

Fifth.—As the next step (Fig. 3), the depth values given by the four upper curves in Fig. 2 have been plotted against frequency for 1 440, 2 880, 4 320, and 5 760-min. durations and the resulting curves extrapolated through the 10-year to the 5-year frequency.

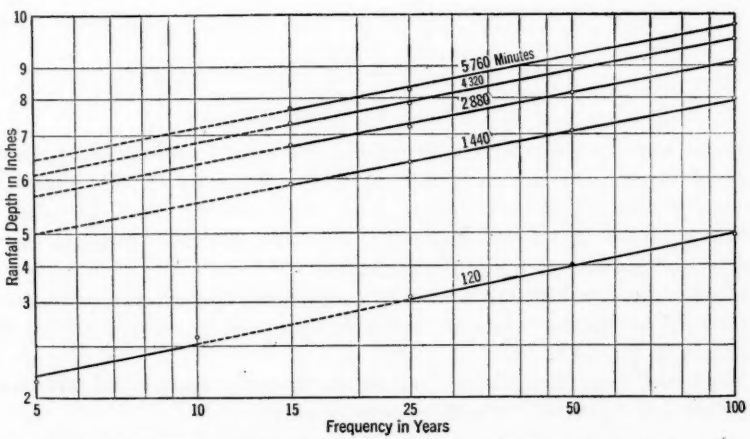

FIG. 3.—CONSTRUCTION AND EXTENSION OF RAINFALL CURVES, QUADRANGLE 15-*E*.

Sixth.—The resulting depth values for 5 and 10-year frequencies were plotted in Fig. 2, giving the two lower curves.

When it is realized that the upper and lower limits of these curves are established by separately prepared data, the writer believes the close agreement of points and curves in all of the eighty quadrangles studied to be unusual.

Seventh.—The slope of each curve in the group was measured carefully and the values for depth, corresponding to the time intervals, 500 min. and 2 000 min., were noted. From data given in Table 2 values of C and n were computed.

TABLE 2.—VALUES FOR C AND n, QUADRANGLE 15-E.

F.	$1-n$.	500^{1-n}.	d.	c_1.	$2\,000^{1-n}$.	d.	c_2.	c.	C.	n.
5	0.24	4.444	3.03	0.682	6.198	4.28	0.692	0.687	41.22	0.76
10	0.23	4.176	3.60	0.862	5.744	4.93	0.858	0.860	51.60	0.77
15	0.23	4.176	3.83	0.917	5.744	5.25	0.914	0.915	54.90	0.77
25	0.23	4.176	4.21	1.008	5.744	5.73	0.998	1.003	60.18	0.77
50	0.23	4.176	4.83	1.157	5.744	6.58	1.146	1.151	69.06	0.77
100	0.23	4.176	5.49	1.313	5.744	7.52	1.311	1.312	78.72	0.77

Eighth.—The record sheet for each quadrangle was completed by noting the depth and intensity formulas for each frequency. These formulas, for Quadrangle 15-E are given in Table 3.

TABLE 3.—RAINFALL DEPTH AND INTENSITY FORMULAS, QUADRANGLE 15-E.

Frequency.	Depth.	Intensity.	Frequency.	Depth.	Intensity.
5	$d = 0.687\ t^{0.24}$	$i = \frac{41.22}{t^{0.76}}$	25	$d = 1.003\ t^{0.23}$	$i = \frac{60.18}{t^{0.77}}$
10	$d = 0.860\ t^{0.23}$	$i = \frac{51.60}{t^{0.77}}$	50	$d = 1.151\ t^{0.23}$	$i = \frac{69.06}{t^{0.77}}$
15	$d = 0.915\ t^{0.23}$	$i = \frac{54.90}{t^{0.77}}$	100	$d = 1.312\ t^{0.23}$	$i = \frac{78.72}{t^{0.77}}$

PREPARATION OF CHARTS

The writer has selected, as a means of presenting the results of this study, a series of charts comparable with the isopluvial charts in the Miami Conservancy report. These charts will make the work of practical value, permitting a simple and rapid method of selecting intensity formulas for any locality within the area under consideration.

Lines of equal value were located by considering the variation to be uniform between the numerical values at the quadrangle centers. The resulting charts are shown in Figs. 4 to 15.

The "iso-C" and "iso-n" lines fell naturally into contours and nowhere was it necessary to disregard isolated or erratic values.

CHARACTERISTICS

The Morgan isopluvial charts, while adaptable to many uses, do not present the relation of rainfall intensity to duration and frequency in a form best suited for small stream-flow analysis or agricultural drain design. Frequency, in a particular problem, becomes a constant, being fixed by the conditions, leaving intensity to vary as, $\frac{C}{t^n}$. In the application of the rational method, t

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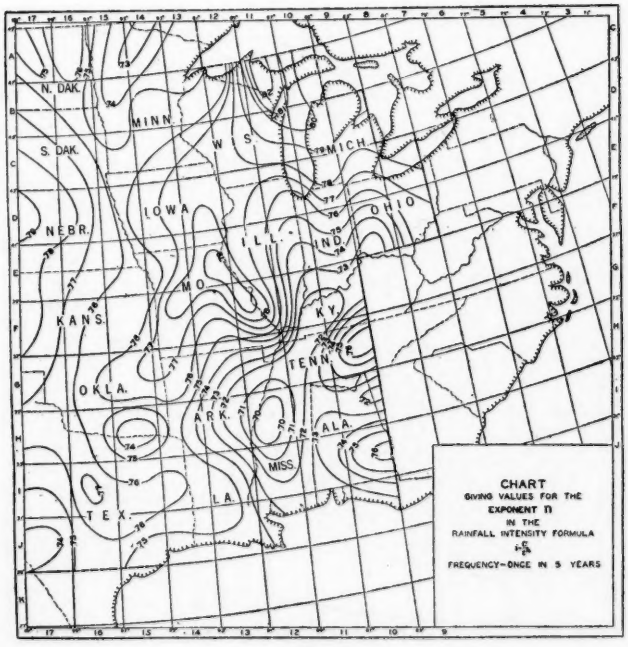
FREQUENCY—ONCE IN 5 YEARS

FIG. 4.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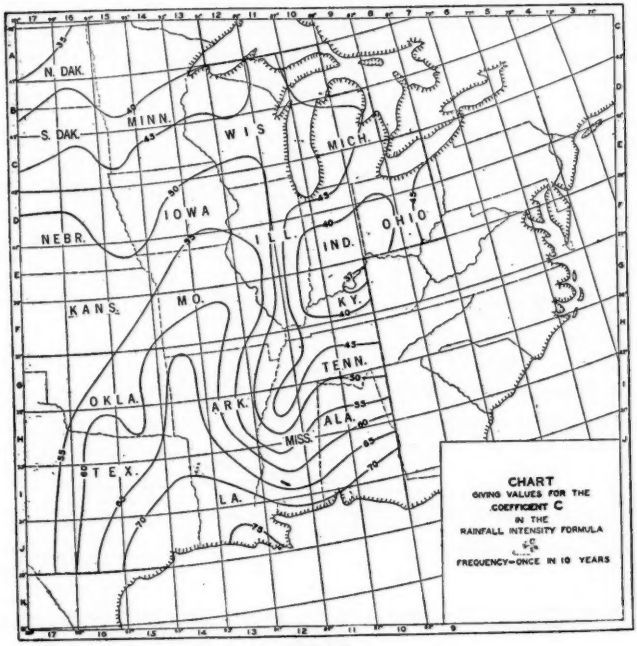
FREQUENCY—ONCE IN 5 YEARS

FIG. 5.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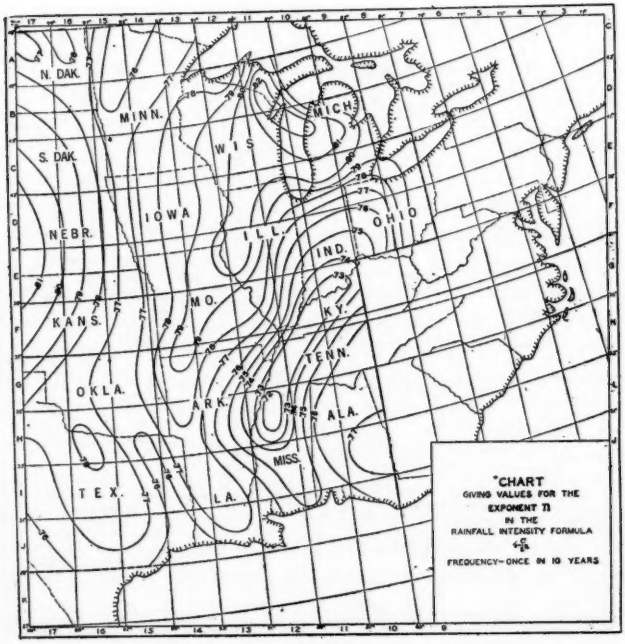
FREQUENCY-ONCE IN 10 YEARS

FIG. 6.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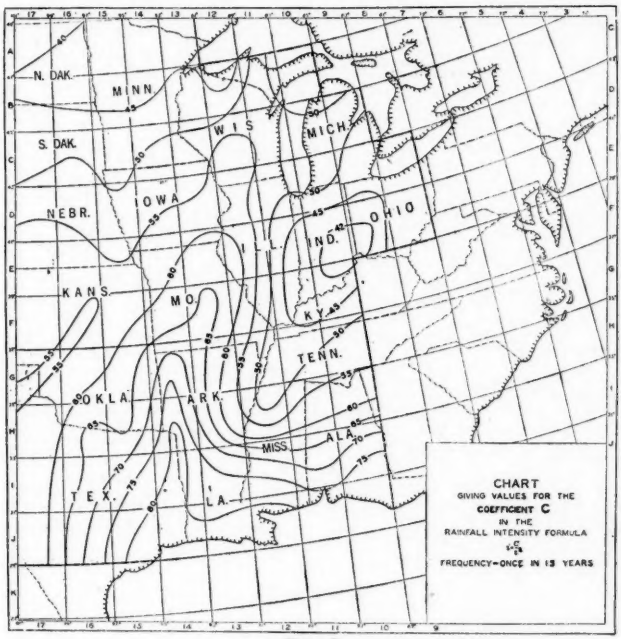
FREQUENCY-ONCE IN 10 YEARS

FIG. 7.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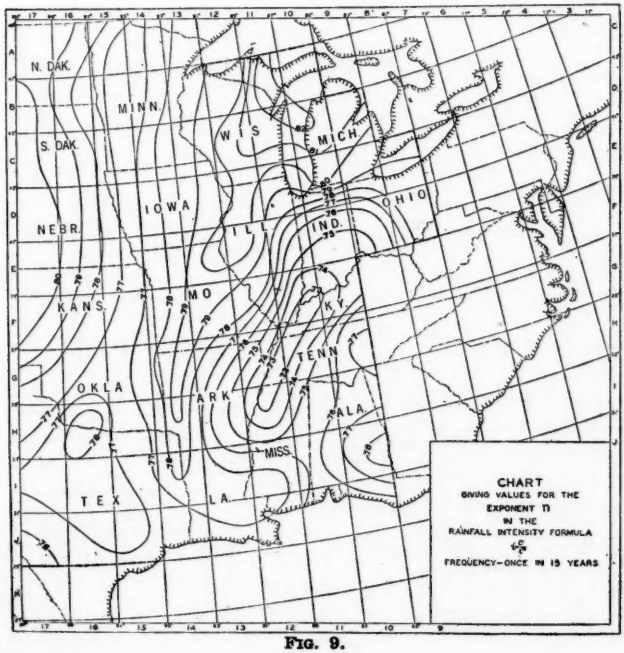
i = C/t^n
FREQUENCY—ONCE IN 15 YEARS

FIG. 8.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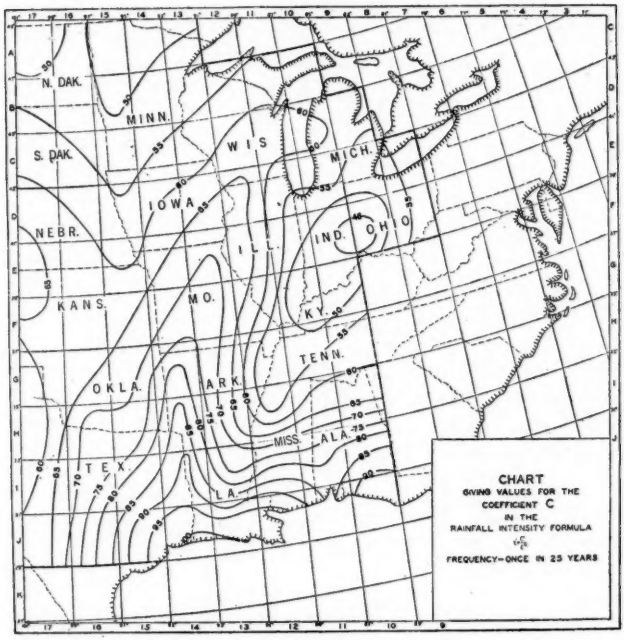
FREQUENCY—ONCE IN 15 YEARS

FIG. 9.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FREQUENCY-ONCE IN 25 YEARS

FIG. 10.

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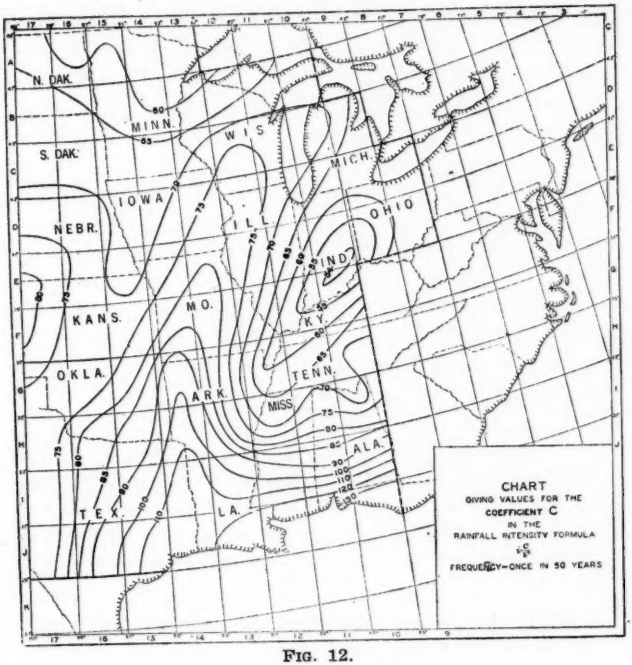
FIG. 11.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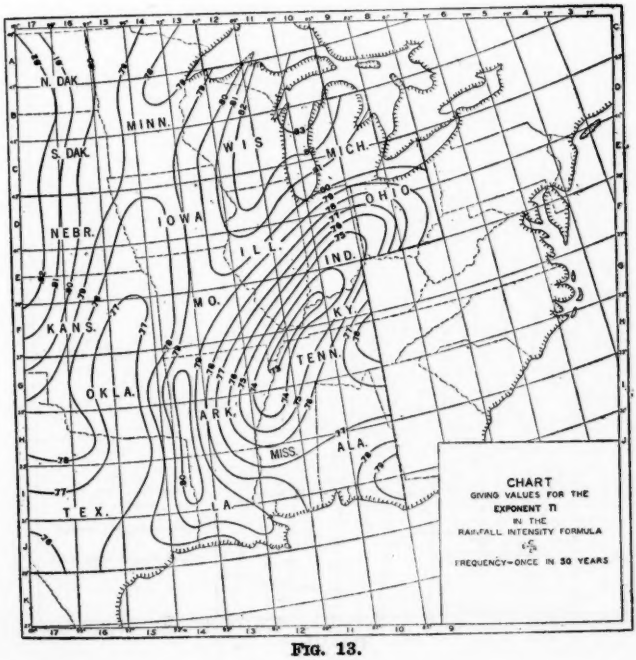
FREQUENCY—ONCE IN 50 YEARS

FIG. 12.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FREQUENCY—ONCE IN 50 YEARS

FIG. 13.

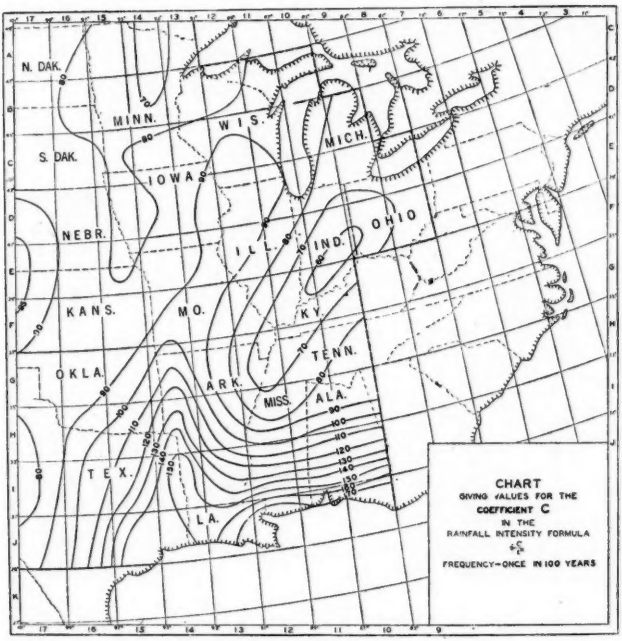
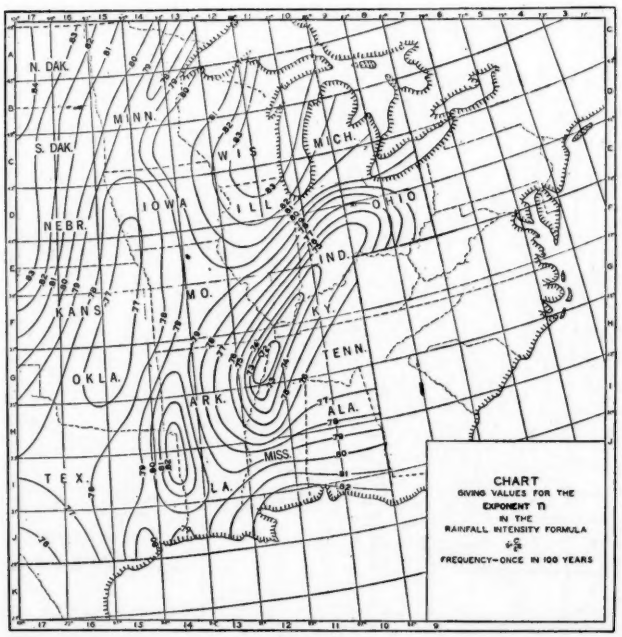

N. DAK.
MINN.
S. DAK.
IOWA
NEBR.
KANS.
OKLA.
TEX.
LA.
MO.
ARK.
MISS.
ILL.
WIS.
MICH.
IND.
OHIO
KY.
TENN.
ALA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
FREQUENCY—ONCE IN 100 YEARS

FIG. 14.


N. DAK.
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S. DAK.
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NEBR.
KANS.
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TEX.
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ARK.
MISS.
ILL.
WIS.
MICH.
IND.
OHIO
KY.
TENN.
ALA.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
FREQUENCY—ONCE IN 100 YEARS

FIG. 15.

will take all values determined by concentration periods and the relationship between time and intensity must be expressed so as to make the proper value of i quickly available for any value of t. The writer's charts are the Morgan isopluvial charts thus interpreted and extrapolated to greater frequencies and shorter duration periods, until they meet and conform with the Meyer intensity curve groups.

The effect of altitude on rainfall intensity is apparent in the Morgan isopluvial charts. Upon comparing the writer's exponential charts with a contour map of the area it is seen that the value of the exponent apparently reflects and measures the effect of altitude. It will be found that, while both values of C and n vary with altitude, they combine in the formula to give the lower intensities in the higher altitudes. A refinement of about 5% can be gained by interpolating values of n to thousandths.

Conclusion

It is to be noted particularly that the general formula, using values from the accompanying charts, is to apply to duration periods greater than 2 hours and is not intended for use within the limits of the Meyer curve groups.

The charts are subject to the limitations of the basic data, which are fully set forth in the references presenting them. The writer believes, however, that he has translated into terms of utility, covering a range of rainfall duration periods not now provided for, the results of two valuable researches and has suggested a new and practical means for the further study of rainfall intensity and its relation to the length of storm period, frequency of occurrence, and locality.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

PAPERS AND DISCUSSIONS

This Society is not responsible for any statement made or opinion expressed in its publications.

FILTERING MATERIALS FOR WATER AND SEWAGE WORKS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION*

The principal items given attention by the Committee on Filtering Materials during 1929 include a further study and development of the sodium sulfate soundness test by an extended series made with the co-operation of thirteen Materials Testing Laboratories; the continuation of tests on sand for rapid sand filters; and a consideration of the question of the proper scope or extent of the investigations to be undertaken and covered in a final report of the Committee.

At the start of the work of the Committee the outstanding questions, or at least the questions which appeared to be directly responsible for the organization of the Committee, related to the problem of selection of materials for sewage trickling filters as regards types and suitability from the viewpoint of their physical characteristics, particularly durability.

Some consideration was given early in the work of the Committee to the question of scope of the investigations to be covered. The first progress report (1925)† indicated something as to what the scope of studies might include. However, as the question of tests and methods of selecting suitable materials appeared to be the more pressing and as it was deemed best not to spread the work of the Committee over too wide a field, its major efforts to date have been devoted to consideration of matters pertaining to testing and selection of trickling filter material as regards physical characteristics.

During the past year (1929) as many as eight or more engineers have prepared discussions of the Committee's work for publication by the Society. Others have made more informal observations. These discussions indicate many phases of the subject of filter materials for water and sewage works, which could be studied extensively. The problem of what to include and where to stop becomes rather pertinent.

* Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1930.

† *Proceedings*, Am. Soc. C. E., March, 1926, Society Affairs, p. 128.

During 1929 the question of scope of the work of the Committee was referred to the Executive Committee of the Sanitary Engineering Division, and a joint meeting of the Executive Committee with representatives of the Committee on Filtering Materials was arranged at Milwaukee, Wis., in July.

As the result of the consideration by the Executive Committee it was concluded that the proper scope for the work of the Committee on Filtering Materials should be in the field of water and sewage works in its broad outlines. It would include questions concerning the relation of physical or qualitative properties of materials, such as durability, sizes, variation in sizes, surface and shape characteristics, cleanness, etc., to the question of quantities as determined by relative loadings, depths of filter beds, thickness of layers, etc. The Executive Committee suggested that the activities of the Committee should be oriented along the following lines:

1.—The ultimate goal of the investigations should be a final report which after proper consideration and modification might become the basis of a "Manual of Engineering Practice" on the subject of filter materials for water and sewage works.

2.—The investigations should include complete consideration of the subject of filter materials for both water and sewage work, with as close co-operation as possible with other agencies, such as the American Water Works Association.

3.—The results of the various studies should be made available as promptly as possible, presumably by means of progress reports.

4.—A semi-final report should be planned for completion by the end of 1930, covering the subject of methods of testing and selecting of filter materials for sewage trickling filters.

The various progress reports of the Committee to date have been oriented on the basis of recording various data, observations, and tentative conclusions covering the current studies so that these data and indicative information may be available without delay for the use of interested engineers. It is hoped that this plan will make useful data quickly available, and also will permit a shorter and more useful final report in that the basic detail data will have been already included in progress reports and interpreted to some extent by a number of engineers.

During 1929 there have been two regular meetings of the Committee, both extending over two days. The first, February 1 and 2, was held at Kansas City, Mo., and included a conference with the Sanitary Engineers of the State Boards of Health of Kansas and Missouri, with reference to tentative experimental station projects being planned for certain of the State Universities. The second meeting was held at Cleveland, Ohio, October 28 and 29, in connection with which an inspection was made of the trickling filters at the Southerly Cleveland, Akron, Canton, and Alliance sewage treatment plants. Both meetings were attended by all members of the Committee.

Other activities of the Committee are briefly indicated in the following paragraphs relating to studies of special phases of the Committee work.

Coarse-Grained Filter Materials for Sewage Works

This topic included mainly the consideration of the feasibility of establishing a series of laboratory tests for the purpose of developing a soundness test for pre-determining the comparative merits of various proposed materials as to probable durability along lines outlined in the 1928 report.* The Committee had tentatively concluded that the sodium sulfate test was the most likely means of pre-determining the probable durability of materials, and a tentative laboratory procedure for making the soundness tests had been developed and tried out by the Committee and by a number of testing engineers in studies extending over a period of three or more years, including the series of tests reported in 1929.† These series of tests indicated that the proposed laboratory procedure for the sodium sulfate treatment for soundness could be used with concordant results when the tests were made in a given laboratory.

To check the test procedure and the reliability of the results when such tests were made in several materials testing laboratories, thirteen laboratories were selected, distributed from Baltimore, Md., and Washington, D. C., on the east to Manhattan, Kans., on the west.

Unfortunately, some of the laboratories apparently did not carefully read or clearly understand the instructions. One laboratory frankly declined to follow the suggested procedure. The results of the co-operative tests were not entirely concordant, due in part to: (1) Variations possible under the laboratory procedure by insufficient control of certain items, such as the matter of temperature; (2) the unavoidable variations in quality of test samples; and (3) deviations from the laboratory procedure. The results of these laboratory tests are given in Appendix I.

After reviewing the results of the co-operative tests and the several discussions and comments from testing engineers, the Committee feels that the sodium sulfate test has value as a practical means of predetermining the comparative merits as to probable durability of several proposed materials for sewage trickling filters. Some modifications in the details of the test procedure may be desirable.

A further comparison of the results of the sodium sulfate soundness test with the effect on the durability of material under actual operating conditions was made by extending the test from twenty to fifty cycles on samples of material taken from five trickling-filter plants where substantially satisfactory operation had been experienced over a term of ten years, or more. This included the beds at Baltimore, Fitchburg, Mass., Gloversville, N. Y., Plainfield, N. J., and Rochester, N. Y. From these extended tests it appears that the destructive effect of the sodium sulfate treatment is continuous and apparently progressive. For practical purposes it appears reasonable to conclude that twenty cycles of the sodium sulfate treatment should be sufficient for comparing the relative soundness of several proposed samples of material. (Appendix II.)

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, p. 1799.
† *Loc. cit.*, Appendix I, p. 1804 *et seq.*

In the 1928 report, reference was made to a projected experimental trickling-filter installation at the Canton Sewage Treatment Plant.* This project is still in the development stage.

Fine-Grained Filter Material for Sewage Works

Studies relating to fine-grained filter material for sewage works have been confined to the literature on the subject, particularly to the reported results of experimental investigations by the Massachusetts State Department of Health and to correspondence with interested engineers.

Filter Materials for Water-Works

The program, as outlined in previous progress reports,† of the study of filter sand, with reference to size and depth, by means of small glass testing filters, has been continued with the co-operation of the water filtration departments of several cities. Certain modifications have been made as to test procedure. A number of the co-operating cities have been quite active and have obtained some interesting results. Others of the cities appear to be interested, but have been unable to operate the test units largely because of insufficient operating personnel. The Committee is assured that a number of the cities are sufficiently interested to proceed with this study.

A conference of interested filtration men and water-works engineers with three members of the Committee was held during the American Water Works Association Convention in Toronto, Ont., Canada, at which a lively discussion of the filter sand studies occurred. The conclusion of this meeting appears to indicate that the series of tests were of value. However, some modification of procedure was suggested, particularly with reference to the use of some form of rate controller and a reduction in the amount of laboratory work, so that the experiments could be handled more easily by the co-operating water departments.

Appendices

Data in some detail are submitted herewith in the form of Appendices in order to make the information available to interested engineers. The subject matter of these Appendices is indicated by title, as follows:

I.—An Analysis of the Results of a Co-Operative Study of the Sodium Sulfate Soundness Test by Thirteen Materials Testing Laboratories on Selected Samples of Limestone and Slag.

II.——Results of Fifty Cycles of the Sodium Sulfate Soundness Test.

Summary

It appears that the scope of the investigation of filter materials should include questions relating to material sizes, relationship of filter material to loadings and filter depths, and the effect of the character of the applied sewage, as well as the matter of physical characteristics with reference to durability.

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, p. 1800.

† *Loc. cit.*, p. 1802, and April, 1928, Society Affairs, p. 272.

This involves a rather broad field of possible study which cannot be covered satisfactorily in a short time.

The Committee hopes to be able during 1930 to outline more definitely the main topics to be included in a final report and the principal difficulties in the use of filter materials which should be given further consideration. Suggestions and comments from members of the Society along these lines would be appreciated.

Respectfully submitted,

WILLIAM E. STANLEY, *Chairman*
JAMES W. ARMSTRONG,
W. H. DITTOE,
GEORGE B. GASCOIGNE,
N. T. VEATCH, JR.

January 16, 1930.

APPENDIX I

AN ANALYSIS OF THE RESULTS OF A CO-OPERATIVE STUDY OF THE SODIUM SULFATE SOUNDNESS TEST BY THIRTEEN MATERIALS TESTING LABORATORIES ON SELECTED SAMPLES OF LIMESTONE AND SLAG

INTRODUCTION

One of the characteristics of a material which largely affects its usefulness as a filter medium in sewage trickling filters is its durability under service conditions. A number of different laboratory tests have been used in an effort to predetermine the probable disintegration of a proposed material. The more frequently used of these tests include either (1) an actual freezing and thawing procedure; or (2), a sodium sulfate treatment that involves subjecting the samples to a process of alternate drying and soaking in a sodium sulfate solution. The freezing and thawing procedure requires considerable time; therefore, a more accelerated test by the use of a solution of sodium sulfate, or some other salt, has been tried in an effort to reduce the time element.

As indicated in previous progress reports,* a considerable experience with the testing of filter materials, extending over three years or more, led the Committee to conclude that the sodium sulfate soundness test was the most likely for indicating the comparative merits of a number of proposed filter materials with reference to probable disintegration. The laboratory procedure for the sodium sulfate test, however, has not been standardized; nor has the test had particularly wide application until recently. Several brief descriptions of laboratory procedures for using sodium sulfate solutions have been suggested, including one by the American Association of State Highway Officials,† which has been used somewhat by various State highway testing

* Particularly the 1928 Report, Appendix II, *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, pp. 1837–1838.

† *Bulletin 1216*, U. S. Dept. of Agriculture, 1924, p. 8.

stations. No method had been developed for measuring the results of the test, except by the personal judgment of the laboratory man, prior to the work done under the direction of this Committee.

On the basis of considerable experience gained in the laboratory of George B. Gascoigne, M. Am. Soc. C. E., and in a number of other laboratories, with use of the sodium sulfate test in various forms, a tentative laboratory procedure was developed. This procedure was rather definitely tried out for the Committee in connection with the series of laboratory tests on samples of trickling filter materials from sixteen service filters which were reported in the 1928 progress report.*

An analysis is given in the following paragraphs of the results of a more extended series of co-operative tests using the sodium sulfate treatment applied to selected material samples.

Purpose of Co-Operative Tests

In order to establish the practicability and reliability of the sodium sulfate soundness test for general use and also to check up the proposed laboratory procedure, the Committee obtained the co-operation of thirteen testing laboratories to conduct a series of tests on carefully selected samples of limestone and slag. It was expected that this series would determine what agreement of test results could be obtained by several laboratories working independently, but using the same laboratory procedure and samples of material of as nearly the same character as it was possible to secure.

Selection and Shipping of Samples

Limestone and slag samples were furnished in duplicate series to each laboratory. The samples were selected so that there were ten pieces in each series, representing five varieties of material. Thus, each series of samples contained duplicate pieces of each variety of material. There were also duplicate series of both limestone and slag samples, each laboratory receiving four series of samples of ten pieces each. An attempt was made to select material samples with varying grades as to durability qualities.

Limestone Samples.—The limestone samples were selected by F. W. Jones, Engineer with George B. Gascoigne, Cleveland, Ohio, and Dr. Herbert F. Kriege, in charge of tests for the France Stone Company Laboratories, at Toledo, Ohio. The material was selected from three different quarries and was reduced from large pieces to the required sizes of test pieces, in order that all pieces of each variety might be as nearly alike as possible.

Five types of stone were selected as follows:

Brown banded, marked "1 and 2" (questionable as to soundness).
Uniform brown, marked "3 and 4" (questionable as to soundness).
Mottled hard, marked "5 and 6" (sound material).
Dense, flinty, marked "7 and 8" (sound material).
Mottled, soft, marked "9 and 10" (unsound material).

In this classification, judgment as to anticipated quality was based on previous experience with material from the several quarries. A duplicate

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, p. 1804.

set of samples was similarly marked 1_a, 2_a, etc. Each of the laboratories was sent a carefully packed box containing the two sets of samples, or twenty pieces in all.

Slag Samples.—The samples of slag were selected and shipped by Fred Hubbard, Consulting Engineer for the Standard Slag Company, Youngstown, Ohio, and Director of the Bessemer Limestone and Cement Company Laboratory, at Bessemer, Pa., who described the material sent out as follows:

"The individual pieces of material have been designated by numbers, 1 to 10, inclusive, and duplicate samples designated by numbers, 1_a to 10_a, inclusive. This material has been selected from five sources as follows:
"Skull No. 1, marked 'Slag 1, 2, 1_a and 2_a.'
Skull No. 2, marked 'Slag 3, 4, 3_a and 4_a.'
Commercial material dense, marked 'Slag 5, 6, 5_a and 6_a.'
Commercial material light, marked 'Slag 7, 8, 7_a and 8_a.'
Skull No. 3, marked 'Slag 9, 10, 9_a and 10_a.'"

These samples of slag were packed carefully in individual round pasteboard cartons and shipped to each laboratory.

Procedure for Making Tests

Rather detailed instructions were given for conducting the tests. The co-operating laboratories were requested to follow the outlined test procedure as closely as possible on one series of each limestone and slag sample and to use the duplicate series of samples for such other tests as they wished. The instructions, together with a detailed description of the test procedure, were published in Appendix II of the 1928 progress report.* The test instructions were not set up as being a finally accepted procedure. It was suggested, however, that they be followed, so that the results by the various laboratories could be readily compared. Criticisms and comments relative to modifications in the procedure were encouraged.

Co-Operating Laboratories

The list of co-operating testing laboratories includes a rather wide range with regard to geographical location and to classification as to principal interests of the several laboratories. The list, including names of engineers and the location of the co-operating laboratories, together with an outline of their classification as to principal interests, was published in the 1928 progress report.† Each of the laboratories very kindly completed the program of tests and reported the results in detail with some discussion of its work. Many of the laboratories extended the investigation to include additional tests and alternate procedures for the sodium sulfate test and, also, in certain cases, freezing and thawing tests.

In the following analysis of the results of the tests by the various laboratories, it has been deemed advisable to designate the laboratory by key letters which do not follow the same sequence as the list of laboratories given in the 1928 progress report.

Test Procedure Used by the Various Laboratories

Before taking up in detail the results of the co-operative soundness tests, it might be well to consider the test procedure used by the several co-operating

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, pp. 1839–1841.
† *Loc. cit.*, pp. 1839 and 1843.

laboratories and note the deviations from that suggested by the Committee. Certain of the laboratories included a discussion of laboratory procedure and made recommendations regarding modifications. These will be referred to later.

The following paragraphs indicate how closely the tests on the primary series of samples followed the procedure set up by the Committee. Some of the laboratories subjected the duplicate set of samples to other soundness tests. Subsequent reference to these tests will be made.

Laboratory A.—Apparently the suggested procedure was followed at this laboratory.

Laboratory B.—The procedure as outlined by the Committee was considered to be susceptible to two interpretations at Laboratory B. Consequently, the following procedure was followed:

"1.—The sample was first dried and weighed as prescribed.
"2.—A saturated solution of sodium sulfate was prepared by dissolving sodium sulfate crystals in water heated to 90° Fahr.
"3.—Samples were immersed in the solution while it was at a temperature of 90° Fahr. and were kept in the solution for 20 hours at room temperature.
"4.—Samples were removed and heated for 4 hours at 105° cent. in a drying oven.
"5.—At the end of 4 hours the samples (still hot) were re-immersed in the sodium sulfate solution saturated at 90° Fahr. and regulated to that temperature when specimens were re-immersed.
"6.—This cycle was repeated until completion of the test."

This deviated from the suggested procedure in that the sodium sulfate solution was raised to 90° Fahr. and saturated at such temperature before each immersion. This quite likely increases the severity of the test to such an extent that it virtually becomes a serious deviation from the proposed procedure.

Laboratory C.—This laboratory was rather frank in stating that the procedure as submitted was not satisfactory. Therefore, it declined to proceed in accordance with the plan suggested by the Committee. The test samples were put through a sodium soundness test, described as follows:

"The samples were dried for 6 hours in an electric oven maintained at 110° cent. They were then introduced, without cooling, into a 15% solution of sodium sulfate, specific gravity 1.140, at a temperature of 21° cent. The specimens were kept in the solution for 18 hours, then dried at 110° cent. for 6 hours, and the cycle was repeated 20 times. The Na_2SO_4 solution was kept in a room, the temperature of which was maintained constantly at 21° cent. $\pm$ 0.5° cent."

The procedure deviated materially from that proposed. This laboratory further declined to record weights of samples before and after the soundness tests, indicating the results only by text.

Laboratory D.—The procedure suggested by the Committee for one series of the test samples was followed at Laboratory D.

Laboratory E.—It was stated by this laboratory that,

"Your routine was followed as closely as possible. During the immersion period the samples were kept in an incubator that maintained a constant tem-

perature of 22 degrees. The drying was perhaps not entirely satisfactory in that the oven was too small to permit good circulation and therefore the temperature at all times was not uniform."

Laboratory F.—The procedure recommended by the Committee was followed here, except that:

"In preparing the specimens * * * the larger pieces were broken up with a hand hammer to maintain more nearly uniformly-sized pieces. In order to eliminate possibility of sharp corners and projections being broken off in handling during test, the edges of all specimens were ground on a carborundum wheel."

Laboratory G.—The method as outlined by the Committee was followed at Laboratory G.

Laboratory H.—The tests were conducted in accordance with the procedure suggested by the Committee, although the laboratory disagreed regarding the question of temperature.

Laboratory I.—It was stated that the "methods of testing were in strict accordance with the procedure as outlined by the Committee." However, the report further indicated that the test procedure was somewhat modified to the extent of placing the samples in wire baskets before immersing them and of keeping the solutions at constant temperature and in covered containers, the stone samples being placed in one jar and the slag samples in another.

Laboratory J.—The procedure suggested by the Committee was followed here, except that when the test samples were removed at the end of each cycle of twenty hours they were examined and then "rinsed in a mild solution of sodium sulfate before being placed in the drying oven." It will be noted that the results obtained by this laboratory show consistently lower losses in the breaking down of the stone than from other laboratories. This is possibly explained by the rinsing before drying which would be likely to diminish the quantity of sodium sulfate contained in the specimen and thus lessen the destructive action.

Laboratory K.—This laboratory undertook to follow the test procedure as suggested by the Committee, but made slight modifications in that wire baskets as sample containers were used, and the samples were also examined after drying so probably they were not replaced in the solution as quickly as they would have been otherwise.

Laboratory L.—The procedure suggested by the Committee was followed at Laboratory L. A maximum and minimum thermometer indicated some variations in room temperature near the solution pans.

Laboratory M.—The Committee's procedure was followed here, except that instead of placing all ten pieces in a large container a small container was used for each piece.

All the laboratories, with the exception of Laboratory C, attempted to comply with the request of the Committee to use the suggested laboratory test procedure. Laboratories B and J made rather radical changes in the procedure, possibly by error. Laboratory F made changes in the shape and sizes of the test pieces, while Laboratories J and K used wire trays and modified the temperature conditions slightly.

TABLE 1.—CO-OPERATIVE SODIUM SULFATE SOUNDNESS TESTS:

Laboratory.	Cycle of Sodium Sulfate Soundness Test.									
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Piece No. 1.										
A	O K	O K	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
B	O K	O K	O K	O K	O K	O K	O K	*b*	*c*	*c*
C	....	....	....	.....	.....	.....	.....	.....	.....	.....
D	O K	O K	O K	O K	O K	O K	O K	Scaled	Scaled	Scaled
E	O K	O K	O K	O K	Slight surface scale	Slight surface scale	Slight increase	Slight increase	Check on one corner	Check on one corner
F	O K	O K	O K	O K	O K	O K	O K	O K	Flaking	Flaking
G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
H	O K	O K	O K	O K	O K	Cracking with grain	O K	O K	O K	O K
I	O K	O K	O K	O K	O K	O K	O K	Split	Split	Split
J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
L	O K	O K	O K	O K	O K	O K	O K	O K	O K	Vertical corner crack
M*	O K	O K	O K	O K	O K	1.5 Sc F	10 Sc F 49 A R	6 H	5 H	O K
Piece No. 2.										
A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
B	O K	O K	O K	*f*	*f*	*c*	*c* + *e*	*d*	*F* + *h*	.....
C	...	.....	.....	.....	.....	.....	.....	.....	.....	.....
D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
E	O K	O K	Surface scaling	Surface scaling	Surface scaling	Surface scaling	Slight crumbling	Slight crumbling	Split in two pieces	Split in three pieces
F	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
H	O K	O K	O K	O K	O K	Cracking	O K	O K	O K	O K
I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
M*	O K	O K	O K	O K	O K	O K	O K	O K	2 H V	O K
Piece No. 3.										
A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
B	O K	O K	O K	O K	O K	O K	O K	*b*	*b*	*b*
C	....	....	....	....	....	....	....	....	....	....
D	O K	O K	O K	O K	O K	O K	O K	O K	O K	Scaled
E	O K	Small scale	Surface scaling	Surface scaling	Surface scaling	Surface scaling	Surface scaling	General crumbling	Progressive	Progressive
F	O K	O K	O K	O K	O K	O K	Slight flake	Slight flake	Slight flake	Slight flake
G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
H	O K	O K	O K	O K	Chipping	O K	O K	O K	O K	O K
I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
L	O K	O K	O K	O K	O K	O K	Corner crack starting	Corner crack starting	Corner crack starting	Corner crack starting
M*	O K	O K	O K	O K	O K	O K	O K	1 H V	O K	O K

*Key to symbols of Laboratory M is given later in the text, p. 1882. "O K" represents no change during the given cycle. This laboratory reported the results of duplicate samples together; thus data for Piece No. 1 include Pieces Nos. 1 and 2; data for Piece No. 2 include Pieces Nos. 1(*a*), and 2(*a*), etc.

Symbols: Laboratory B.

a = chipping and spalling.
b = fine cracks.
c = increase in cracking.
d = severely cracked, wide or deep.
e = broken into three or more pieces.
f = cracked perpendicular to bedding plane.
F = failure.
g = no increase in cracking or chipping.
h = no future test.

PROGRESSIVE OBSERVATIONS ON LIMESTONE SAMPLES.

Laboratory.		Cycle of Sodium Sulfate Soundness Test.									
		11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Piece No. 1.	A	Spalling *c*	Spalling *c*	Spalling *c* + *f*	Spalling *F*	Spalling *F* + *h*	Spalling	Spalling	Spalling	Spalling	Spalling
	B										
	C										
	D	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled
	E	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner	Check on one corner
	F	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking
	G	Cracking	Cracking	Cracking	Cracking	Cracking	Cracking	Cracking	Cracking	Cracking	Failed
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	Split	Failed								
	J	O K	Small piece off	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	Vertical corner crack	Vertical corner crack	Vertical corner crack	Slight spall	Slight spall	Slight spall	Slight spall	Slight spall	Slight spall	Slight spall
	M*	45 H	O K	20 V	O K	8 H	O K	O K	O K	5 F	13 F
Piece No. 2.	A	Spalling	Spalling	Spalling	Spalling	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured
	B										
	C										
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase
	F	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking	Flaking
	G	O K	O K	O K	O K	O K	O K	Small edge crack	Small edge crack	Small edge crack	O K
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	1 Sc F	O K	10 H Sc	O K	2 equal pieces, A R	12 H Sc	O K	23 H V	1 Sc	147.5 H A
Piece No. 3.	A	Spalling *a* + *b*	Spalling *a* + *b*	Spalling *g*	Spalling *c*	Spalling *d*	Spalling *d*	Spalling *d*	Spalling *d*	Spalling *d*	Spalling *F*
	B										
	C										
	D	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled	Scaled
	E	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive
	F	Slight flake	Slight flake	Slight flake	Slight flake	Slight flake	Slight flake	Crumbling	Crumbling	Crumbling	Crumbling
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	Small piece off	Small piece off	Small piece off	Small piece off	Small piece off	Small piece off	Small piece off	Small piece off
	L	Corner crack starting	Corner crack starting	Corner crack starting	Corner crack starting	Corner crack starting	Two pieces spalled	Two pieces spalled	Two pieces spalled	Two pieces spalled	Two pieces spalled
	M*	9 Sc	2 Sc	44 Sc V	21 H Sc	O K	4 H Sc	14 H Sc	11 S c	45 H V Sc	25 H V Sc A

TABLE 1.—

Piece	Laboratory.	Cycle of Sodium Sulfate Soundness Test.									
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Piece No. 4.	A	O K	O K	O K	O K	O K	Spalling	Spalling	Spalling	Spalling	Spalling
	B	O K	O K	O K	O K	O K	O K	*b*	*a* + *c*	*F* + *h*	.....
	C	....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	Scaled
	E	O K	O K	Slight surface scaling	Slight surface scaling	Slight surface scaling	Split in two pieces	Split in six pieces	Twelve, or more pieces	Crumbled	No further test
	F	O K	O K	O K	O K	Slight flake	Slight flake	Slight flake	Slight flake	Slight flake	Slight flake
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	Chipping	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	O K	O K	O K	O K	O K	O K	4 H V	O K	O K	O K
Piece No. 5.	A	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	B	O K	O K	O K	*b*	*g*	*g*	*g*	*c*	*g*	*g*
	C	....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	O K	O K	O K	O K	Slight scaling	Slight scaling	Slight scaling	Slight scaling	Slight scaling	No increase
	F	O K	O K	O K	O K	O K	O K	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	Split	Split	Split	Split
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	Fine corner crack	0.8 gramme piece off	O K	O K	O K
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
Piece No. 6.	A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
	B	O K	*a*	*g*	*a*	*a*	*g*	*g*	*a*	*g*	*a*
	C	....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	O K	O K	O K	O K	O K	O K	Slight flaking	Slight flaking	Slight flaking	No increase
	F	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	Small piece off	O K
	L	O K	O K	O K	O K	O K	Fine corner crack	No change	No change	No change	Crack enlarging
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K

(*Continued.*)

	Laboratory	Cycle of Sodium Sulfate Soundness Test.									
		11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Piece No. 4.	A	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
	B	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	Scaled	Scaled	Scaled	Two pieces	Two pieces	Two pieces	Two pieces	Four pieces	Five pieces	Five pieces
	E	No further test									
	F	Slight flake	Slight flake	Slight flake	Slight flake	Crumbled	Crumbled	Crumbled	Crumbled	Crumbled	Crumbled
	G	O K	O K	O K	O K	Chipped	Chipped	Chipped	Chipped	Chipped	Chipped
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	J	O K	O K	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake	Surface crumble or flake
	K	O K	O K	O K	O K	O K	O K	Small piece off	Small piece off	Small piece off	Small piece off
	L	Slight spall cracks	Slight spall cracks	Slight spall cracks	Slight spall cracks	Slight spall cracks	Slight spall	Slight spall	Slight spall	Two fine cracks	Two fine cracks
	M*	10 Sc	2 Sc	4 Sc	10 Sc	2 Sc	8 Sc	12 Sc	11 Sc V	12 Sc V	16.5 Sc V
Piece No. 5.	A	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	B	*g*	*g*	*g*	*g*	*c*	*c*	*c*	*c*	*d*	*d* + *a*
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase
	F	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked
	G	O K	O K	O K	O K	O K	O K	O K	O K	Chipped	O K
	H	Small cracks	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	Split	Split	Split	Split	Split	Failed	.....	.....	.....	.....
	J	O K	Small piece off	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	4 H V	O K	O K	O K	6 H V	O K	O K	O K	O K	O K
Piece No. 6.	A	Spalling	Spalling	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured
	B	*g*	*g*	*g*	*g*	*g*	*g*	*g*	*g*	*g*	*g*
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase
	F	O K	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked	Slightly flaked
	G	O K	O K	O K	Broken	Broken	Broken	Broken small pieces	Broken small pieces	Broken small pieces	Failed
	H	Small cracks	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	Split	Failed	.....	.....	.....	.....
	J	O K	Small piece off	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	Little change	Little change	Little change	1 gramme piece off	Large crack	28.3 gramme piece off	6.4 gramme piece off	Spall	2.5 gramme piece off	2.3 gramme piece off
	M*	O K	O K	O K	O K	O K	22 H V	O K	O K	O K	O K

TABLE 1.—

Piece	Laboratory	Cycle of Sodium Sulfate Soundness Test.									
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
No. 7.	A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
	B	O K	O K	O K	*d* + *f*	*c* + *e*	*c*	*c*	*d*	*d*	*d*
	C	....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	E	O K	O K	O K	O K	O K	O K	Slight flaking	Slight flaking	Split in two pieces	Split in two pieces
	F	O K	O K	O K	O K	Crack open	Flake	Flake	Two pieces	Two pieces	Two pieces
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	Cracking	O K	O K
	I	O K	O K	O K	O K	O K	O K	Split	Split	Split	Split
	J	O K	O K	O K	Vertical crack	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
No. 8.	A	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	B	O K	O K	O K	O K	O K	O K	*b*	*d*	*d*	*d*
	C	....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	O K	Checked	Checked
	E	O K	O K	O K	O K	O K	O K	Slight flaking	Slight flaking	Slight flaking	No increase
	F	O K	O K	O K	O K	O K	O K	Slight flaking	Slight flaking	Slight flaking	Two pieces
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	Cracking	O K	O K
	I	O K	O K	O K	O K	O K	O K	Split	Split	Split	Split
	J	O K	O K	O K	Vertical crack	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
No. 9.	A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Fractured	Fractured	Fractured
	B	O K	*d*	*d*	*F*	*F*	*F*	*F* + *h*	.....	.....	.....
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	O K	O K	O K	O K	O K	Checked	Five pieces	Five pieces
	E	O K	Partial split	General crumbling	Progressive	Split in four pieces	Split in four pieces	12 or more pieces	Progressive crumbling	Progressive crumbling	Progressive crumbling
	F	O K	O K	O K	O K	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	Cracking	O K	O K	O K	O K	O K	O K	O K
	I	O K	O K	O K	O K	O K	O K	Failed	Crumbled	Crumbled	Crumbled
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	Showed cracking at end of fourth cycle with continued cracking and surface disintegration to end; cracked into small pieces						
	L	O K	Cracking	2 pieces split	Split many pieces	.....	.....	.....	.....	.....	.....
	M*	5 H V	12 H V	50 H V	14 H V	98 D H V	23 V	O K	0.5 H V	O K	O K

(*Continued.*)

Piece	Laboratory.	Cycle of Sodium Sulfate Soundness Test.									
		11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
No. 7.	A	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling
	B	*d*	*F* + *h*	.....	.....	.....	.....	.....	.....	.....	.....
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	O K	O K	Checked	Checked	Checked	Checked	Checked	Checked	Two pieces	Two pieces
	E	Split in two pieces	Split in three pieces	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase
	F	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces
	G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	Split	Split	Split	Split	Split	Failed	.....	.....	.....	.....
	J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	Slight spall	O K
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
No. 8.	A	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	B	*d* + *f*	*F* + *h*	.....	.....	.....	.....	.....	.....	.....	.....
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	Checked	Checked	Checked	Checked	Checked	Checked	Two pieces	Four pieces	Six pieces	Six pieces
	E	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase	No increase
	F	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces	Two pieces
	G	O K	Cracked in two	Cracked in two	Broken smaller pieces	Broken smaller pieces	Broken smaller pieces	Broken smaller pieces	Broken smaller pieces	Broken smaller pieces	Failed
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	Split	Failed	.....	.....	.....	.....	.....	.....	.....	.....
	J	O K	Small piece off 0.5 gramme	O K	O K	O K	O K	O K	O K	O K	O K
	K	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	L	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	M*	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
No. 9.	A	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured
	B	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	D	Five pieces	Five pieces	Eight pieces	Eight pieces	Eight pieces	Twenty-two pieces	Twenty-two pieces	Twenty-two pieces	Twenty-seven pieces	Twenty-seven pieces
	E	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling	Progressive crumbling
	F	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface
	G	O K	O K	O K	O K	O K	O K	O K	O K	Chipped, otherwise O K	
	H	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
	I	Failed	.....	.....	.....	.....	.....	.....	.....	.....	.....
	J	O K	Small piece off	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling
	K	Showed cracking at end of fourth cycle with continued cracking and surface disintegration to end; cracked into small pieces									
	L	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	M*	O K	14 H	O K	O K	O K	O K	O K	O K	O K	O K

TABLE 1.—

Piece No. 10. Laboratory.	Cycle of Sodium Sulfate Soundness Test.									
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
A	O K	Spalling	Spalling	Spalling	Spalling	Spalling	Spalling	Fractured	Fractured	Fractured
B	O K	*d*	*d*	*F*	*F*	*F*	*F* + *h*	.....	.....	.....
C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
D	O K	O K	O K	O K	O K	O K	O K	Nine pieces	Twelve pieces	Twelve pieces
E	O K	Partial split	General crumbling	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive
F	O K	O K	O K	O K	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface
G	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
H	O K	O K	Cracking	O K	O K	O K	O K	O K	O K	O K
I	O K	O K	O K	O K	O K	O K	O K	Split	Split	Failed
J	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K
K	O K	O K	O K	Showed cracking at end of fourth cycle with continued cracking and surface disintegration during remainder of twenty cycles; two large pieces						
L	O K	Cracking	Cracking	29.3 gramme off	Crumbling	Crumbling	9.7 gramme off	Blew up	.....	.....
M*	O K	1 H V	25 H V	20 H V	9 H V	2.5 H V	4 H V	13 H V	O K	2 H V

Tabulated Test Results

The observations of the several co-operating laboratories were reported in accordance with the form suggested by the Committee with two exceptions, in which the principal report included text comments. Twelve of the thirteen laboratories reported weights. Laboratory C stated that weights were regarded as misleading and declined to make any weighings.

The results of the observations were presented in two classes: First, the progressive observations of the condition of the test pieces for each cycle of the sodium sulfate application; and, second, the change in weights. The progressive observations of the effect of the test on the limestone samples are given in Table 1. The data on weights are given in Tables 2 and 3 for the limestone samples and in Table 4 for the slag samples.

The changes in condition of the slag samples were so slight that it has been considered unnecessary to include the progressive observations as to the condition of these samples. The weights indicate that for 67% of the slag samples, the weights at the end of the tests were greater than at the beginning.

Comparative Analysis of Test Results—Limestone Samples

An attempt was made in selecting the test pieces—especially the limestone—to obtain some samples of sound material, some samples of definitely unsound material, and other samples of a questionable quality somewhere between the two extremes.

A study of the results submitted by the several laboratories shows that, with few exceptions, the slag samples withstood the sodium sulfate treatment

(*Continued.*)

Piece No. 10. Laboratory.	Cycle of Sodium Sulfate Soundness Test.									
	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
A	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured	Fractured
B	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
C	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
D	Fourteen pieces	Fourteen pieces	Fifteen pieces	Fifteen pieces	Fifteen pieces	Twenty-six pieces	Twenty-six pieces	Twenty-seven pieces	Thirty-one pieces	Thirty-one pieces
E	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive	Progressive
F	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface	Crumbled on surface
G	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
H	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
I	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
J	OK	Small piece off	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling	Surface crumbling
K	Showed cracking at end of fourth cycle with continued cracking and surface disintegration during remainder of twenty cycles; two large pieces									
L	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
M*	15 H V 65 A R	12 H V	1 H	OK	OK	OK	OK	OK	OK	OK

with little evidence of deterioration. Piece No. 6 was reported by Laboratories A and D as fracturing in the nineteenth cycle. Most of the observations in which any change was noted had to do with slight spalling or fine cracks. In one or two instances, the sharp edges of the porous pieces were reported as exhibiting signs of weathering.

Several of the investigators reported difficulty in bringing the pieces of slag back to their original weights. This was most notable in the porous material. The explanation of this may possibly lie in a chemical union of the sodium sulfate with some constituents of the slag.

As the sodium sulfate treatment on the slag samples produced very little effect, these tests are of less usefulness in checking the practicability of the suggested test procedure than the results of the tests on limestone.

The test data on limestone samples (Tables 1, 2, 3, and 5), show a rather wide range in the results found by the several laboratories. However, the general averages of the results in terms of the percentage losses in weight appear to classify the several samples of material roughly into sound, unsound, and questionable material, in agreement with the original selection of samples. This is illustrated by the percentage loss of weight data in Table 5. The wide variation in results is also illustrated by this table.

The several series of test samples included five varieties as regards quality of material, with two duplicate pieces of each variety, making ten pieces in all. Some indication of the agreement which can be expected from tests made in a given laboratory on the same material may be obtained by considering

TABLE 2.—Co-Operative Sodium Sulfate Soundness Tests.
Change in Weights After Twenty Cycles on Limestone Samples.

Laboratory.	Weights, in Grammes.				
	Weight at start.	After Twenty Cycles.			
		Weight of pieces more than 1 in.	Loss, percentage.	Weight of pieces less than ¼ in.	Loss, percentage.
	Piece No. 1.				
A	305.5	281.0	8.0	19.0	6.2
B	240.5	34.0	86.0	70.5	29.0
C	No weights reported.				
D	203.0	184.0	9.4	19.0	9.4
E	278.4	276.4	0.7	2.0	0.7
F	160.0	150.6	5.9	9.4	5.9
G	171.0	159.0	7.0	.....	.....
H	306.9	295.9	3.6	9.0	2.9
I	267.0	173.0	35.0	49.0	18.5
J	194.0	193.7	0.2	0.3	0.2
K	178.9	178.9	0.0	0 0	0.0
L	337.0	336.0	0.3	1.0	0 3
M	325.0	234.0	28.0	26.0	8.0
	Piece No. 2.				
A	197.0	155.0	21.4	26.0	13.2
B	222.5	0.0	100.0	106.5	48.0
C	No weights reported.				
D	228.0	228.0	0.0	0.0	0.0
E	286.7	235.2	17.9	29.5	10.0
F	204.8	204.0	0.4	0.8	0.4
G	191.0	188.0	1.6	.....	.....
H	370.1	367.0	0.8	1.5	0.4
I	366.0	373.0	.....	.....	.....
J	208.4	207.9	0.2	0.5	0.2
K	186.4	184.6	1.0	1.8	1.0
L	176.0	176.0	0.0	0.0	0.0
M	416.5	301.0	27.6	50.0	12.0
	Piece No. 3.				
A	208.5	200.0	4.1	5.0	2.4
B	202.5	140.0	31.0	18.5	9.0
C	No weights reported.				
D	198.0	186.0	6.1	12.0	6.1
E	272.4	106.8	60.7	93.3	34.0
F	161.8	138.0	14.6	23.6	14.6
G	246.0	231.0	6.1	.....	.....
H	252.6	240.4	4.8	12.2	4.8
I	265.0	277.0	.....	.....	.....
J	174.1	173.6	0.3	0.5	0.3
K	167.8	160.6	4.3	3.9	2.3
L	237.0	233.0	1.7	4.0	1.7
M	452.0	289.0	36.1	123.0	27.2

Laboratory.	Weights, in Grammes.				
	Weight at start.	After Twenty Cycles.			
		Weight of pieces more than 1 in.	Loss, percentage.	Weight of pieces less than ¼ in.	Loss, percentage.
	Piece No. 4.				
A	204.0	199.0	2.5	4.0	2.0
B	254 5	0.0	100.0	151.5	60.0
C	No weights reported.				
D	202.0	166.0	17.8	28.0	13.9
E	291.0	0.0	100.0	154.6	53.0
F	163.6	84.2	48.6	20.2	12.4
G	231.0	213.0	7.7	.....	....
H	213.8	189.3	11.5	22.3	10.4
I	292.0	304.0	.....	.....	....
J	171.5	170.7	0.5	0.8	0.5
K	170.5	158.8	6.9	3.7	2.2
L	274.0	270.0	1.5	4.0	1.5
M	338.5	247.0	27.0	85.0	25.1
	Piece No. 5.				
A	245.5	244.0	0.6	1.5	0.6
B	243.0	224.0	7.8	5.0	2.0
C	No weights reported.				
D	220.0	220.0	0.0	0.0	0.0
E	143.4	138.7	3.3	4.7	3.3
F	216.2	212.4	1.9	4.2	1.9
G	250.0	234.0	6.4	....	....
H	239.7	237.6	0.9	2.1	0.9
I	230.0	214.0	7.0	16.0	7.0
J	291.6	291.5	0.1—	0.1	0.1—
K	268.2	267.1	0.4	1.1	0.4
L	215.0	214.0	0.4	1.0	0.4
M	603.0	593.0	1.6	0.0	0.0
	Piece No. 6.				
A	227.0	186.0	18.1	2.5	1.1
B	204.5	187.0	8.6	7.5	3.7
C	No weights reported.				
D	300.0	300.0	0.0	0.0	0.0
E	249.0	246.5	1.0	2.5	1.0
F	218.3	219.0	0.0	0.0	0.0
G	240.0	205.0	14.6	....	....
H	240.0	238.6	0.6	1.4	0.6
I	240.0	234.0	2.5	6.0	2.5
J	186.2	186.1	0.1—	0.1	0.1—
K	218.1	217.0	0.5	1.1	0 5
L	184.0	143.0	22.3	0.0	0.0
M	335.0	334.0	0.3	0.0	0.0

TABLE 2.—(*Continued.*)

Laboratory.	Weights, in Grammes.				
	Weight at start.	After Twenty Cycles.			
		Weight of pieces more than 1 in.	Loss, percentage.	Weight of pieces less than ¼ in.	Loss, percentage.
	Piece No. 7.				
A	216.0	202.0	6.5	0.5	0.2
B	195.5	104.0	46.7	10.5	5.4
C	No weights reported.				
D	177.0	176.0	0.6	1.0	0.6
E	170.7	168.8	1.0	1.5	0.8
F	242.0	233.3	3.6	1.2	0.5
G	243.0	236.0	2.9	....	....
H	169.0	168.8	0.1	0.2	0.1
I	252.0	207.0	17.8	6.0	2.4
J	146.0	146.0	0.0	0.0	0.0
K	244.5	244.5	0.0	0.0	0.0
L	176.0	175.0	0.1	1.0	0.1
M	339.5	339.5	0.0	0.0	0.0
	Piece No. 8.				
A	175.5	174.0	0.8	1.0	0.6
B	213.0	121.0	43.0	5.0	2.3
C	No weights reported.				
D	203.0	199.0	2.0	4.0	2.0
E	167.7	166.5	0.7	0.4	0.2
F	282.6	282.5	0.0	0.0	0.0
G	242.0	235.0	2.9	....	....
H	166.4	155.4	6.6	7.6	4.6
I	219.0	192.0	12.3	14.0	6.4
J	193.4	192.7	0.4	0.7	0.4
K	270.0	270.0	0.0	0.0	0.0
L	130.0	130.0	0.0	0.0	0.0
M	404 0	404 0	0.0	0.0	0.0

Laboratory.	Weights, in Grammes.				
	Weight at start.	After Twenty Cycles.			
		Weight of pieces more than 1 in.	Loss, percentage.	Weight of pieces less than ¼ in.	Loss, percentage.
	Piece No. 9.				
A	244.5	0.0	100.0	44 5	18.2
B	312.5	0.0	100.0	256.5	82.0
C	No weights reported.				
D	226.0	122.0	46.0	40.0	17.7
E	251.0	20.6	92.0	128.5	51.0
F	186.4	183.8	1.4	2.6	1.4
G	209.0	196.0	6.2	.....	....
H	287.1	143.6	50 0	43.3	15.1
I	286.0	54.0	81.2	77.0	26.9
J	250.8	241.4	3.8	9.4	3.8
K	360.2	258.7	28.2	44.2	12.3
L	147.0	0.0	100.0	24.0	16.3
M	354.0	311.0	12.1	9.0	2.5
	Piece No. 10.				
A	294.0	0.0	100.0	17.5	5.9
B	346.0	0.0	100.0	290.3	84 6
C	No weights reported.				
D	245.0	103.0	58.0	37.0	15.1
E	238.7	78.2	67.0	89.7	37.5
F	231.0	226.0	2.2	5.0	2.2
G	254.0	236.0	7.1	.....	....
H	232.7	173.8	25.3	24.2	10.4
I	245.0	0.0	100.0	67.0	27.3
J	228.7	220.4	3.6	8.3	3.6
K	319.0	308.2	3.4	10.8	3.4
L	137.0	0.0	100.0	30.0	22.0
M	454.5	375.0	17.6	35.0	7.7

the results of the tests on the duplicate pieces of each variety of material. Further, an approximate indication of the concordance of results from various laboratories can thus be obtained, also.

Through the efforts of Mr. Jones, a careful analysis has been made of the results of the several tests at the end of twenty cycles as given by the observations recorded in Table 1 and also by the loss in weight of pieces of less than 1 in. from Table 2, and a concordance rating has been determined for each of the five series of duplicate test pieces. In considering these results, the tests by Laboratory C have been disregarded because the procedure used by this laboratory was a definite deviation from that used by the other laboratories. Further, Laboratory C furnished no weight measurements.

The analysis of the findings of the twelve laboratories at the end of the twentieth cycle on Pieces Nos. 1 and 2, as given in Tables 1 and 2, indicated fairly concordant results from each individual laboratory on the duplicate pieces with the exception of Laboratories E and I. Considering all the results

of the twenty-four tests by twelve different laboratories (which included twenty-four pieces of presumably the same material), sixteen of the results appeared to be in close agreement and the other eight in fair agreement; but, whereas the effect of the test on the first sixteen pieces was practically negligible, the effect on the remaining eight pieces was rather disastrous. This is indicated particularly by the weight losses and also by photographs of the test pieces at the end of twenty cycles. For convenience in evaluating the soundness test as regards securing comparable results on similar materials, it was considered that the findings on Pieces Nos. 1 and 2 indicated an agreement rating of about 67 per cent.

TABLE 3.—CO-OPERATIVE SODIUM SULFATE SOUNDNESS TESTS.
TOTAL WEIGHT LOSSES IN LIMESTONE SAMPLES AT END OF TWENTY CYCLES.

Laboratory.	Weight, in Grammes.				
	At start of test.	After Twenty Cycles.			
		Weight of pieces of more than 1 in.	Loss less than 1 in., percentage.	Weight of pieces less than ¼ in.	Loss less than ¼ in., percentage.
A	2 317.5	1 641.0	29.2	121.5	5.3
B	2 434.8	810.0	66.8	921.0	37.8
C	No weight results reported				
D	2 202.0	1 884.0	14.4	141.0	6.4
E	2 349.0	1 437.7	38.7	506.7	21.6
F	2 066.5	1 933.8	6.4	67.1	3.2
G	2 277.0	2 133.0	6.3	(Not furnished)	
H	2 478.3	2 210.4	10.8	123.8	5.0
I	2 662.0	1 997.0	25.0	235.0	8.8
J	2 044.7	2 024.0	1.0	20.7	1.0
K	2 383.6	2 248.4	5.7	65.5	2.7
L	2 013.0	1 677.0	16.7	65.0	3.2
M	4 022.0*	3 427.5*	14.8	328.0	8.1
Totals	29 250.4	23 423.8	.....	2 595.3†	.....
Averages	.....	.....	19.9	.....	9.3

* Double samples, including the duplicate set also.
† Weights by Laboratory G not included. Total at start would be 26 973.4 grammes.

In a similar manner ratings have been determined for the other duplicate test pieces. The several concordant or agreement ratings for the five sets of duplicate pieces obtained from this analysis are, as follows:

	Percentage.
Pieces Nos. 1 and 2	67
Pieces Nos. 3 and 4	58
Pieces Nos. 5 and 6	83
Pieces Nos. 7 and 8	75
Pieces Nos. 9 and 10	71
Average	71

This average rating of 71% represents the conclusion of one investigator based in a large measure on judgment factors rather than mathematical aver-

TABLE 4.—Co-Operative Sodium Sulfate Soundness Tests. Change in Weights after Twenty Cycles on Slag Samples

Laboratory.	Weight, in Grammes.				
	At start.	After Twenty Cycles.			
		Pieces more than 1 in.	Loss, percentage.	Pieces less than ¼ in.	Loss, percentage.
Test Piece No. 1.					
A	226.0	226.0	.....	0.0	.....
B	296.0	298.2	*	0.0	.....
C	.....	.....	.....	.....	.....
D	270.0	270.0	.....	0.0	.....
E	231.5	231.5	.....	0.0	.....
F	210.8	212.7	*	0.0	.....
G	192.0	193.0	*	0.0	.....
H	246.6	248.3	*	0.0	.....
I	198.0	200.0	*	0.0	.....
J	227.2	229.2	*	0.0	.....
K	283.0	285.0	*	0.0	.....
L	133.0	136.0	*	0.0	.....
M	529.0	519.0	1.9	0.0	0.0
Test Piece No. 2.					
A	209.0	209.0	.....	0.0	.....
B	300.5	301.2	*	0.0	.....
C	.....	.....	.....	.....	.....
D	176.0	176.0	.....	0.0	.....
E	315.5	315.7	.....	0.0	.....
F	248.4	250.9	*	0.0	.....
G	242.0	242.0	.....	.....	.....
H	253.4	255.6	*	0.0	.....
I	202.0	204.0	*	0.0	.....
J	180.5	181.7	*	0.0	.....
K	191.3	192.4	*	0.0	.....
L	167.0	169.0	*	0.0	.....
M	308.0	308.0	.....	0.0	.....
Test Piece No. 3.					
A	215.0	215 0	.....	0.0	.....
B	308.0	315.0	*	0.0	.....
C	.....	.....	.....	.....	.....
D	239.0	232.0	2.9	7.0	2.9
E	199.0	198.7	0.2	0.0	.....
F	127.8	131.7	*	0.0	.....
G	191.0	195.0	*	0.0	.....
H	189.2	196.3	*	0.0	.....
I	209.0	210.0	*	0.0	.....
J	170.9	174.0	*	0.0	.....
K	151.3	155.0	*	0.0	.....
L	196.0	202.0	*	0.0	.....
M	356.0	356.0	.....	0.0	.....

Laboratory.	Weight, in Grammes.				
	At start.	After Twenty Cycles.			
		Pieces more than 1 in.	Loss, percentage.	Pieces less than ¼ in.	Loss, percentage.
Test Piece No. 4.					
A	218.0	218.0	.....	0.0	.....
B	373.5	368.8	.....	4 7	1.3
C	.....	.....	.....	.....	.....
D	226.0	226.0	.....	0.0	.....
E	178.5	182.0	*	0.0	.....
F	184.3	190.2	*	0.0	.....
G	152.0	154.0	*	0.0	.....
H	183.0	192.3	*	0.0	.....
I	176.0	178.0	*	0.0	.....
J	246.4	253.7	*	0.0	.....
K	225.5	229.3	*	0.0	.....
L	170.0	172.0	*	0.0	.....
M	427.5	427.5	.....	0.0	.....
Test Piece No. 5.					
A	235.0	235.0	.....	0.0	.....
B	241.5	248.4	*	0.0	.....
C	.....	.....	.....	.....	.....
D	202.0	202.0	.....	0.0	.....
E	276.4	276.4	.....	0.0	.....
F	190.6	193.1	*	0.0	.....
G	183.0	184.0	*	0.0	.....
H	209.5	213.6	*	0.0	.....
I	202.0	202.0	*	0.0	.....
J	231.5	234.0	*	0.0	.....
K	168.4	171.2	*	0.0	.....
L	198.0	200.0	*	0.0	.....
M	439.5	439.5	.....	0.0	.....
Test Piece No. 6.					
A	152.0	152.0	.....	0.0	.....
B	226.5	225.5	.....	1.0	0.4
C	.....	.....	.....	.....	.....
D	283.0	283.0	.....	0.0	.....
E	188.4	189.0	*	0.0	.....
F	256.0	257.7	*	0.0	.....
G	180.0	181.0	*	.....	.....
H	222.0	223.5	*	0.0	.....
I	251.0	256.0	*	0.0	.....
J	262.6	265.0	*	0.0	.....
K	270.0	272.0	*	0.0	.....
L	189.0	189.0	.....	0.0	.....
M	528.5	528.5	.....	0.0	.....

* Final weights greater than initial weights and could not be reduced by continued boiling in water, for 80 out of a total of 120 reports giving weights (67%).

TABLE 4.—(*Continued*).

Laboratory.	Weight, in Grammes.				
	At start.	After Twenty Cycles.			
		Pieces more than 1 in.	Loss, percentage.	Pieces less than ¼ in.	Loss, percentage.
Test Piece No. 7.					
A	84.5	84.5	....	0.0	.....
B	165.0	163.5	.....	1.5	0 9
C	.....	.....	.....	.....	.....
D	94.0	94.0	.....	0.0	.....
E	117.8	120.8	*	0.0	.....
F	90.8	90.8	.....	0.0	.....
G	120.0	124.0	*	0.0	.....
H	174.0	176.0	*	0.0	.....
I	136.0	119.0	12.5	.....	.....
J	158.3	160.2	*	0.0	.....
K	171.8	174.1	*	0.0	.....
L	113.0	115.0	*	0.0	.....
M	288.5	288.5	.....	0.0	.....
Test Piece No. 8.					
A	135.0	136.0	*	0.0	.....
B	175.5	180.0	*	0.0	.....
C	.....	.....	.....	.....	.....
D	150.0	150.0	.....	0.0	.....
E	137.6	139.0	*	0.0	.....
F	91.9	92.8	*	0.0	.....
G	124.0	124.0	.....	0.0	.....
H	176.8	180.4	*	0.0	.....
I	137.0	140.0	*	0.0	.....
J	159.3	165.0	*	0.0	.....
K	198.0	202 9	*	0.0	.....
L	181.0	187.0	*	0.0	.....
M	421.0	421.0	.....	0.0	.....

Laboratory.	Weight, in Grammes.				
	At start.	After Twenty Cycles.			
		Pieces more than 1 in.	Loss, percentage.	Pieces less than ¼ in.	Loss, percentage.
Test Piece No. 9.					
A	139.5	140.0	*	0.0	.....
B	227.0	237.2	*	0.0	.....
C	.....	.....	.....	.....	.....
D	126.0	126.0	.....	0.0	.....
E	230.0	235.8	*	0.0	.....
F	207.4	212.6	*	0.0	.....
G	160.0	163.0	*	0.0	.....
H	146.3	154.3	*	0.0	.....
I	166.0	171.0	*	0.0	.....
J	195.6	202.6	*	0.0	.....
K	130.2	135.8	*	0.0	.....
L	105.0	109.0	*	0.0	.....
M	458.5	458.5	.....	0.0	.....
Test Piece No. 10.					
A	168.0	169.0	*	0.0	.....
B	190.5	198.2	*	0.0	.....
C	.....	.....	.....	.....	.....
D	155.0	155.0	.....	0.0	.....
E	216.0	222.0	*	0.0	.....
F	163.7	167.7	*	0.0	.....
G	150.0	152.0	*	0.0	.....
H	112.8	116.3	*	0.0	.....
I	127.0	127.0	.....	0.0	.....
J	126.3	130.8	*	0.0	.....
K	109.2	112.4	*	0.0	.....
L	126.0	133.0	*	0.0	.....
M	377.0	377.0	.....	0.0	.....

* Final weights greater than initial weights and could not be reduced by continued boiling in water, for 80 out of a total of 120 reports giving weights (67%).

ages or determinations. Accordingly, it is not unlikely that other investigators might arrive at a somewhat different rating with reference to the agreement of the results from the data.

A further study of the test results shows that the most severe tests were reported by the several laboratories, as follows:

Pieces Nos. 1 and 2, by Laboratories A, B, E, I, and M.
Pieces Nos. 3 and 4, by Laboratories B, E, F, and M.
Pieces Nos. 5 and 6, by Laboratories A, G, I, and L.
Pieces Nos. 7 and 8, by Laboratories B, G, and I.
Pieces Nos. 9 and 10, by Laboratories A, B, D, F, H, I, K, L, and M.

This tabulation and a study of the test results and also photographs taken at the end of the tests show that, apparently, Laboratories B, E, and I obtained throughout results which were more severe than those obtained by the other laboratories. The severity of the test as found by Laboratory B may be

influenced by the fact that the material was immersed after each cycle with the solution saturated at 90° Fahr. In Laboratory E, the immersed samples were kept at a constant temperature of 22° cent., which would permit the forming of a large amount of crystals. This, then, may account in part for the rather disastrous effects found. It was reported that the jars of sodium sulfate used by Laboratory I were kept at a constant temperature, although the exact temperature was not reported. If this temperature was about 20° cent., it would support the assumptions made regarding the findings of Laboratory E.

Laboratory J seemed to secure the least effect on any of the pieces tested. The explanation for this may lie in the fact that, in each cycle, when the pieces of material were removed from the sodium sulfate, they were rinsed with a mild sodium sulfate solution before drying.

Alternative Sodium Sulfate Soundness Tests

The instructions to the co-operating laboratories suggested that the duplicate series of test samples might be used as desired. Several of the testing engineers applied a modified form of sodium sulfate test to the duplicate samples. Freezing and thawing tests were applied to the duplicate samples by A. T. Goldbeck and C. H. Scholer, Associate Members, Am. Soc. C. E.

Mr. Goldbeck extended the soundness test investigations to include eight variations of the test procedure on a sample of soft marble for the purpose of considering some of the variables in the test.*

Mr. Hubbard, of the Bessemer Limestone and Cement Company Laboratory, Bessemer, Pa., subjected the duplicate series of test samples to a sodium sulfate soundness test, which he designated as "the A. S. T. M. method".† It was stated by Mr. Hubbard that,

"About the only difference in the two methods [this and the Committee procedure] is that the A. S. T. M. method does not require the solution to be saturated at 86° to 90° F., and it also does not require the pieces to be re-immersed in the solution while hot or immediately upon removal from the oven".

It was further stated that approximately the same results were obtained by the two test procedures.

A. S. Rea, Chief Engineer, of the Bureau of Tests, Department of Highways, Columbus, Ohio, subjected the duplicate series of stone and slag, to a procedure similar to the prescribed test,

"With the exception that in place of drying in an oven for four hours at 105°C., they were allowed to dry in air at room temperature (20°–23°C.) for twenty-four hours between each immersion.

"In the comparison of the results from the prescribed method and the results from the modified method of drying in air it seems that the latter gives chiefly a surface disintegration."

Freezing and Thawing Soundness Tests

Mr. Goldbeck applied a freezing and thawing test to the duplicate set of samples, extending the tests to 100 cycles. Professor Scholer applied a freezing

* *The Crushed Stone Journal*, July, 1929, p. 13.

† *Proceedings*, Am. Soc. for Testing Materials, Vol. 28, 1928, p. 361, Report of Committee C–9. This test procedure has not been adopted as a standard or tentative standard method by the American Society for Testing Materials.

TABLE 5.—Co-Operative Sodium Sulfate Soundness Tests.
Percentage Loss of Material at End of Twenty Cycles—Limestone.

Laboratory.	Limestone Sample Numbers.														
	1.			2.			3.			4.			5.		
	(a).	(b).	(c).	(a).	(b).	(c).	(a).	(b).	(c).	(a).	(b).	(c).	(a).	(b).	(c).
A	1.8	6.2	8.0	8.2	13.2	21.4	1.7	2.4	4.1	0.5	2.0	2.5	0	.6	.6
B*	57.0	29.0	86.0%	52.0	48.0	100%	22.0	9.0	31.0%	40.0	60.0	100.0%	5.8	2.0	7.8%
C†															
D		Error		0	0	0	0	6.1	6.1%	3.9	13.0	17.8%	0	0	0
E	0	0.7	0.7%	7.9	10.0	17.9%	26.7	34.0	60.7%	47.0	53.0	100.0%	0	3.3	3.3%
F	0	5.9	5.9%	0	0.4	0.4%	0	14.6	14.6%	36.2	12.4	48.6%	0	1.9	1.9%
G‡		‡	7.0		‡	1.6		‡	6.1		‡	7.7		‡	6.4
H	0.7	2.9	3.6%	0.4	0.4	0.8%	0	4.8	4.8%	1.1	10.4	11.5%	0	0.9	0.9%
I	16.5	18.5	35.0%	0	0	0	0	0	0	0	0	0	0	0.9	0.9%
J*	0	0.2	0.2%	0	0.2	0.2%	0	0.3	0.3%	0	0.5	0.5%	0	0	0
K	0	0	0	0	0.1	0.1%	2.0	2.3	4.3%	4.7	2.2	6.9%	0	0.4	0.4%
L	0	0.3	0.3	0	0	0	0	1.7	1.7%	0	1.5	1.5%	0	0.4	0.4%
M	20.0	8.0	28.0%	15.6	12.0	27.6%	8.9	27.2	36.1%	1.9	25.1	27.0%	1.6	0	1.6%
Totals	96.0	71.7	174.7	84.1	84.3	170.0	61.3	102.4	169.8	135.3	181.1	324.0	7.4	10.4	23.6
Averages	9.6	7.2	17.5	7.6	7.7	14.2	5.6	9.3	14.1	12.3	16.5	26.0	0.7	0.9	2.0
Averages, omitting Laboratories B and J	4.4	5.3	9.8	3.6	4.0	7.0	4.4	10.3	13.9	10.6	13.4	22.4	0.2	0.9	1.6
Original classification of samples§	Questionable.			Questionable.			Questionable.			Questionable.			Sound.		

Notes.—(a) = Percentage of loss of material between ¼ in. and 1 in. in size; (b) = percentage of loss of material less than ¼ in. in size; (c) = percentage of loss of material less than 1 in. in size.

* These laboratories made rather definite modifications in test procedure.

† This laboratory made no weight measurements.

‡ Report of this laboratory did not indicate amount of losses less than ¼ in.

§ General classification of samples as to soundness, made at time of selection of sample based on previous experience.

‡ Report of this laboratory did not indicate amount of losses less than ¼ in.
§ General classification of samples as to soundness, made at time of selection of sample based on previous experience.

TABLE 5.—(*Continued.*)

Laboratory.	Limestone Sample Numbers.														
	6.			7.			8.			9.			10.		
	(*a*).	(*b*).	(*c*).	(*a*).	(*b*).	(*c*).	(*a*).	(*b*).	(*c*).	(*a*).	(*b*).	(*c*).	(*a*).	(*b*).	(*c*).
A	17.0	1.1	18.1%	6.3	0.2	6.5%	0.2	0.6	0.8%	81.8	18.2	100.0%	94.1	5.9	100.0%
B*	4.9	3.7	8.6%	41.3	5.4	46.7%	41.7	2.3	43.0%	18.0	82.0	100.0%	15.4	84.6	100.0%
C†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
D	0	0	0	0	0.6	0.6%	0	2.0	2.0%	28.3	17.7	46.0%	42.9	15.1	58.0%
E	0	1.0	1.0%	0.2	0.8	1.0%	0.5	0.2	0.7%	41.0	51.0	92.0%	29.5	37.5	67.0%
F	0	0	0	3.1	0.5	3.1%	0	0	0	0	1.4	1.4%	0	2.2	2.2%
G‡	.....	‡	14.6	.....	‡	2.9	.....	‡	2.9	.....	‡	6.2	.....	‡	7.1
H	0	0.6	0.5%	0	0.1	0.1%	2.0	4.6	6.6%	34.9	15.1	50.0%	14.0	10.4	25.3%
I	0	2.5	2.5%	15.4	2.4	17.8%	5.9	6.4	12.3%	54.3	26.9	81.2%	72.7	27.3	100.0%
J*	0	0	0	0	0	0	0	0.4	0.4%	0	3.8	3.8%	0	3.6	3.6%
K	0	0.5	0.5%	0	0	0	0	0	0	15.9	12.3	28.2%	0	3.4	3.4%
L	22.3	0	22.3%	0	0.1	0.1%	0	0	0	83.7	16.3	100.0%	78.0	22.0	100.0%
M	0.3	0	0.3%	0	0	0	0	0	0	9.6	2.5	12.1%	9.9	7.7	17.6
Totals	44.5	9.4	68.7	66.3	10.1	78.8	50.3	16.5	68.7	367.5	247.2	621.9	357.4	219.7	584.2
Averages	4.1	0.9	5.7	6.0	0.9	6.6	4.6	1.5	5.7	33.3	22.4	51.8	32.4	20.0	48.6
Averages, omitting Laboratories B and J results	4.4	0.6	6.0	2.8	0.5	3.2	1.0	1.5	2.5	43.3	17.9	51.8	38.0	14.6	48.1
Original classification of sample§	Sound.			Sound.			Sound.			Unsound.			Unsound.		

*These laboratories made rather definite modifications in test procedure.
† This laboratory made no weight measurements.
‡ Report of this laboratory did not indicate amount of losses less than ¼ in.
§ General classification of samples as to soundness, made at time of selection of sample based on previous experience.

and thawing procedure up to 250 cycles. The tabulated results of these freezing and thawing tests are given in Tables 6 and 7, respectively.

TABLE 6.—Results of Soundness Tests by Freezing and Thawing, Laboratory B.

(100 cycles on both limestone and slag check samples by Goldbeck.)

Sample No.	Weights of Limestone Samples.					Weights of Slag Samples.				
	At start.	At End of Test.		Loss of Weight—Percentage.		At start.	At End of Test.		Loss of Weight—Percentage.	
		Pieces more than 1 in.	Pieces less than ½ in.	Pieces more than 1 in.	Pieces less than ½ in.		Pieces more than 1 in.	Pieces less than ½ in.	Pieces more than 1 in.	Pieces less than ½ in.
1(*a*)	278	277	0	0	0	293	289	.....	1	0
2(*a*)	226	225		0		335	335	.....	0	0
3(*a*)	173	172	0	0	0	230	75	13	78	4
4(*a*)	153	152		0		340	340	.....	0	0
5(*a*)	235	228	1	3	0	182	182	.....	0	0
6(*a*)	258	249		3		269	269	.....	0	0
7(*a*)	184	182	1	1	0	115	115	.....	0	0
8(*a*)	214	205		4		156	154	.....	1	0
9(*a*)	292	118	436	60	72	205	205	.....	0	0
10(*a*)	310	0		100		246	246	.....	0	0

The procedure was described by Mr. Goldbeck as follows:

"The freezing apparatus consisted of a six-compartment ice cream cabinet supplied with a water-cooled compressor. Each compartment is 10 in. in diameter and 20 in. deep. It was partially filled with alcohol which surrounded a galvanized can 6½ in. square and 20 in. deep. This can contained the specimens, each in a separate, small container, and all specimens were totally immersed in water."

The following phenomena are to be noted in connection with Table 6:

Limestone Samples:

(1) All pieces are apparently sound, except Nos. 9(*a*) and 10(*a*).

(2) Pieces Nos. 3(*a*) and 4(*a*) were unaffected by test.

(3) Pieces Nos. 1(*a*) and 2(*a*) showed fine cracks at the forty-fifth cycle, with no change thereafter.

(4) Piece No. 5(*a*) showed chipping and spalling with fine cracks at thirtieth cycle and further chipping at thirty-fifth cycle, with no further change thereafter.

Slag Samples:

(5) All pieces were apparently sound, except No. 3(*a*).

(6) Pieces Nos. 2(*a*), 5(*a*), 6(*a*), (7*a*), 8(*a*), and 9(*a*) were unaffected by test.

(7) Piece No. 1(*a*) showed chipping and spalling at thirtieth cycle with no further change thereafter.

(8) Piece No. 3(*a*) showed chipping and spalling at fifth cycle with severe cracking at tenth cycle and failure at thirtieth cycle.

(9) Piece No. 6(*a*) showed fine cracks at the fifteenth cycle and spalling at fortieth cycle, with no further changes.

(10) Pieces Nos. 7(*a*) and 8(*a*) showed chipping and spalling at thirty-fifth cycle preceded by fine cracks in Piece No. 8(*a*) at thirtieth cycle. There was no further change after the thirty-fifth cycle.

(11) Pieces Nos. 9(*a*) and 10(*a*) showed chipping and spalling at fifth cycle with further chipping and spalling observed at each five cycles, resulting in complete failure at forty-eighth cycle for Piece No. 9(*a*) and at thirty-fifth cycle for Piece No. 10(*a*).

(12) Pieces Nos. 4(*a*) and 10(*a*) showed fine cracks at fifteen and twenty cycles, respectively, with no further change.

TABLE 7.—RESULTS OF SOUNDNESS TESTS BY FREEZING AND THAWING, LABORATORY I.

(250 cycles on both limestone and slag check samples by Scholer.)

LIMESTONE.

	1(*a*).	2(*a*).	3(*a*).	4(*a*).	5(*a*).	6(*a*).	7(*a*).	8(*a*).	9(*a*).	10(*a*).
Weight at beginning	339	266	195	262	170	237	132	124	386	371
Weight after 250 cycles	335	257	191	262	167	230	130	124	286	207
Loss, in grammes	4	9	4	0	3	7	2	0	100	164
Loss, in percentage	1.8	3.4	2.0	0	1.8	3.0	1.5	0	25.9	44.2

Total weight at beginning	2 482
Total weight after 250 cycles	2 189
Total loss, in grammes	293
Total loss, in percentage	11.8

SLAG.

	1(*a*).	2(*a*).	3(*a*).	4(*a*).	5(*a*).	6(*a*).	7(*a*).	8(*a*).	9(*a*).	10(*a*).
Weight at beginning	202	198	219	176	256	213	142	168	121	140
Weight after 250 cycles	200	194	216	174	251	210	127	160	113	133
Loss, in grammes	2	4	3	2	5	3	15	8	8	7
Loss, in percentage	1.0	2.0	1.4	1.3	2.0	1.4	10.6	4.8	6.6	5.0

Total weight at beginning	1 835
Total weight after 250 cycles	1 778
Total loss, in grammes	57
Total loss, in percentage	3.1

"The period of freezing extended over 16 hours and of thawing over 8 hours. This program was varied in a few instances.

"Temperatures as low as —14° Fahr. are possible with this apparatus and the rate of cooling depends upon the load in the apparatus. In general, however, complete freezing of the can of water containing the specimens took place within 8 hours.

"The following readings were taken to determine the change in temperature of the alcohol in the compartments of the freezer. Each of the six compartments contained a concrete specimen 6 in. by 6 in. by approximately 12 in. at

the time the readings were made. Specimens and the water in which they were immersed were at 74° Fahr. at the time of immersion.

"Time period of Immersion.	Temperature, in degrees Fahrenheit.
Before immersing specimen	—8
5 min	4
10 "	8
15 "	12
20 "	14
25 "	14
30 "	15
35 "	15
40 "	15
45 "	15
50 "	14
55 "	14
65 "	12
75 "	11
85 "	11
115 "	10
175 "	8
250 "	4
24 hours	—10"

The freezing and thawing procedure by Professor Scholer was similar to that described by Mr. Goldbeck, except that the test samples were saturated with water but not immersed.

The conclusions of Mr. Goldbeck relative to the comparison between freezing and thawing and sodium sulfate tests were stated to be:

"(*a*) The freezing and thawing tests conducted as described herein were not as severe as the sodium sulfate tests and did not give corresponding results.

"(*b*) In general, it would seem that the sodium sulfate test is more severe than the freezing test.

"(*c*) It is believed that the freezing test offers a much more certain criterion of the durability of aggregates than the sodium sulfate test and particularly if the samples are only partially immersed in water during the freezing operation. Partial immersion seems to give much more rapid results than total immersion."

Professor Scholer's conclusions were that, in general, results of the sodium sulfate and the freezing and thawing tests are in accordance and that the 250 cycles of freezing and thawing at —15° Fahr. gave results closely paralleling the results of the sodium sulfate tests.

The principal objection to freezing and thawing tests for soundness is the length of time required, because of the extended number of cycles of alternative freezing and thawing necessary to produce results comparable to those of twenty cycles of sodium sulfate treatment.

Theory of the Sodium Sulphate Test

There appears to be some uncertainty as to just what takes place during the sodium sulfate test cycle. Two general opinions have been expressed.

1.—During immersion sodium sulfate is absorbed into the pores of the material and during heating in the oven it crystallizes and tends to disrupt the sample.

2.—During immersion sodium sulfate is drawn up into the pores of the material where it crystallizes and exerts a disrupting effect in a manner similar to the freezing of water drawn into the pores of a rock. The heating in the oven drives out any excess water and makes room for further absorption and crystallization during subsequent immersions.

The expressed opinion of investigators seems more frequently inclined toward the second theory. This theory appears to account more satisfactorily for the differences noted due to variation in temperature during the immersion period. If a material is submerged in sodium sulfate solution and the temperature varies so that more or less crystals are formed, it is expected that the disrupting effect will follow directly, the greater disrupting effect occurring when the greater number of crystals are formed.

Mr. Rea states that:

"It was noticed that most of the splitting and disintegration occurred while the stones were in the sulfate solution rather than during the period of drying. This could be explained by the theory that the anhydrous crystals are formed in drying the stone at 105° cent., and when the stone is re-immersed crystals of greater hydration are formed. It has been found possible to disintegrate a stone of poor quality by immersing it in a saturated sodium solution with an excess of the salt present and heating and cooling the solution a number of times."

Mr. Goldbeck observes:

"Although the action of the sodium sulfate test in disrupting the samples may be chemical to a slight extent, it seems to have been established that this action is almost entirely of a mechanical nature due to the growth of sodium sulfate crystals. This being the case, it is obvious that before the sodium sulfate test can be standardized, the variables which affect crystallization must be eliminated."

An excellent statement of the theory of the sodium sulfate test has been prepared for the Committee, by Dr. H. H. King, Head of the Department of Chemistry, Kansas State College, as follows:

"The reactions which take place in the sodium sulfate acceleration soundness test on filter material appear to be as follows:

"To the pore spaces are added succeeding increments of sodium sulfate in solution. In order for the same fractional space to be filled with the salt each time it is essential to use a uniform strength solution. After the pore spaces are filled with solution, the material is dried at 100°; that is, such a temperature as will finally dehydrate the salt completely.

"When the material containing the anhydrous salt is again immersed in the salt solution, it will tend to hydrate to the same form as that existing in the solution. Obviously, the maximum disruptive force comes through forming the highest hydrate, the decahydrate, which happens to be the stable form below 32.48° cent. In order to obtain the formation of the maximum amount of solid decahydrate from the anhydrous salt, it should be brought in contact with a saturated solution in order that it will not partly re-dissolve.

"From these considerations it is clear that the salt solution should be of constant strength and saturated, or nearly so. It must contain the salt dissolved in the form of the decahydrate, and the temperature must be kept constant. In preparing the saturated salt solution, if it is heated above 32.48° cent., the salt both solid and dissolved, will be converted to the anhydrous form. In cooling again supercooling may take place, since crystals

of the decahydrate are no longer present. Under these circumstances the salt would exist in solution as the heptahydrate, which in saturated solution at 21° cent. contains approximately twice as much actual Na_2SO_4 as the saturated decahydrate solution. Since at this temperature (21° cent.) the heptahydrate is unstable and tends to go to the decahydrate at the first opportunity, and since in doing so a considerable amount of solid decahydrate would be crystallized out, this may explain its high disruptive effect noted by one observer.

"However, there is nothing in the solubility data of sodium sulfate in the Int. Crit. Tables (v. 4, p. 236) to indicate that the heptahydrate would be formed by cooling a solution of the decahydrate to 12° cent., or below, as suggested by one observer. The critical point is at 32.48° cent., and solutions should not be saturated above that point. As a matter of fact the solubility curve between 20 and 30° cent. is so steep that one, as a matter of safety, might well make 30° cent. the limit of heating the solution and still obtain rapid solution of Glauber's salt. If it does not appear certain that Glauber's salt is crystallizing out on cooling, a few crystals of Glauber's salt should be added to start it.

"It should be noted that results obtained with a 15% Glauber's salt solution as suggested by one observer, are not comparable to those obtained with a saturated solution, for 100 g. of water will dissolve 46.96 g. of Glauber's salt at 21° cent. Also, making 15% Glauber's salt solution involves some uncertainty depending on the humidity when the salt is weighed out. The decahydrate readily loses water of crystallization under dry conditions.

"Since the salt cannot hydrate above 32.48° cent. the disruptive effect of hydration would be entirely lost if the material were treated in the solution at 37.5° cent. as suggested by one observer."

Temperature Considerations

The temperatures at which the sodium sulfate solution is saturated and maintained and the variations in temperature appear to affect the results of the test materially. Some indications are given in the foregoing discussion of the relation of temperature and the sodium sulfate action.

Sodium sulfate appears to crystallize in at least three forms, depending mainly on temperatures, namely, the decahydrate ($Na_2SO_4.10H_2O$); the heptahydrate ($Na_2SO_4.7H_2O$); and the anhydrous (Na_2SO_4).

It is established that the anhydrous form crystallizes at about 32° or 33° cent., while the decahydrate is crystallized below this temperature, except that there appears to be some question as to what lower temperature the third form, heptahydrate, may be found. Laboratory C stated that only below 12° cent. is the heptahydrate formed. Other studies of this item indicate that the heptahydrate may exist in solution up to 24° cent. Furthermore, since the heptahydrate is unstable it may pass entirely into the decahydrate in solution. If the test is dependent upon the formation of crystals in the container during immersion, particularly below 32° cent., it appears questionable that there would be a more severe action due to the formation of heptahydrate below 12° cent., as suggested by Laboratory C, since the heptahydrate crystals are less voluminous than the decahydrate. It seems more likely that at lower temperatures the disrupting effect, if any, will be more pronounced because of the formation of a larger amount of crystals rather than to the particular type of the crystal.

The observations and comments of the testing engineers indicate that variations in temperature of the solution will produce marked variations in the results obtained. It is generally agreed that it is important that the temperature of the solution during immersion of the samples should be closely controlled. However, there is apparently no unanimity of opinion as to the proper temperature at which to maintain the solution.

It has been suggested that the solution be maintained at some constant temperature between 35° cent. to 45° cent. since, within that range, a small change would cause very little change in the density of the solution.

Messrs. Penniman and Brown, Analytical and Consulting Chemists, Baltimore, Md., consider a fixed temperature very important, and have suggested that a temperature of 37.5° cent. might be more easily obtained than one of 70° Fahr. and should be maintained throughout the tests. Professor Scholer has suggested that the temperature of the solution be automatically maintained at some point between 40° cent. and 50° cent., except that at the end of each immersion period the solution be allowed to cool to 30° cent., in order to crystallize out the more voluminous decahydrate.

Opposed to refinement of the test by closer temperature is the practical consideration of the expense and the more complex apparatus required to maintain a constant temperature, particularly the higher temperatures suggested. It has been suggested that as great or greater variations in the results of soundness tests may be anticipated, due to variations in the quality of the stone from the same source, as will be brought about by temperature changes during the progress of the test.

Laboratory Apparatus

The original instructions of the Committee did not specify the apparatus, except to indicate it in a general way. The comments of certain of the co-operating laboratories indicate that the simpler the apparatus the greater the possibility of securing concordant results. It would appear that the solution containers used should be covered to reduce evaporation to a minimum. There is, also, a suggestion of a possible refinement by the use of separate small containers for each sample piece of material tested, as practiced by Laboratory M. However, one large container can be used effectively, provided the area of the container is large enough to permit each piece to have a definite location without serious interference from any other piece. The practice of Laboratory K of placing in the container a ¼-in. mesh, galvanized-iron wire tray raised about ½ in. above the bottom so that the specimens will will be surrounded by the solution, merits consideration. The use of a 5-gal. jug, as reported by Laboratory I, appears to offer no particular advantage except that evaporation would be at a minimum. The same effect might be accomplished with less bulky containers.

The apparatus required for careful control of temperatures would materially reduce the simplicity of the test. It is still a mooted question as to whether the additional expense and difficulties are warranted.

Measuring and Reporting Results

Prior to the work of the Committee, there appears to have been no effort made to measure the results of the soundness test quantitively. The co-operative tests indicate that the determination of loss by weighting of the sample before and after the sodium sulfate treatment was very satisfactory for the limestone samples. However, the weights for the slag samples were of little value as about 67% of the samples increased in weight.

The selection of the sizes larger than 1 in. and smaller than ¾ in. as the basis of determining losses was somewhat arbitrary. However, no evidence has been produced to indicate that other sizes might be better suited to the testing of sewage filter material.

The form suggested* by the Committee for reporting the results of the soundness test appears in general to have been acceptable. Several of the testing engineers have developed symbols by which to designate the result of any particular cycle of the test. At Laboratory M, Dr. Kriege has developed a rather comprehensive set of symbols for soundness test observations† which he has tried out on a great many tests. These are as follows:

H, H_2, H_3, etc. = Horizontal chips from 1, 2, or 3 pieces, etc.
V, V_2, V_3, etc. = Vertical chips from 1, 2, or 3 pieces, etc.
A, A_3, A_4, etc. = Horizontal split into 2, 3, or 4 pieces, etc.
B, B_3, B_4, etc. = Vertical split into 2, 3, or 4 pieces, etc.
Ban = Banding failure.
C = Imminent checking and splitting.
Ch = Failure around cherty inclusions and limestone contact.
Chv = Failure across cherty inclusions and limestone contact.
Cr = Failure of crystals at contact.
C = Complete disintegration by crumbling, etc.
D_1, D_2, etc. = Number pieces showing complete disintegration by crumbling, etc.
F = Flaking—scaling on any side, horizontal or vertical.
I = Infilling crumbling.
L = Laminations failing.
O K = Passed through individual test cycle "O. K."
R = Broken pieces retained in the series to observe further breaking action.
S = Surface solution effect.
Sc = Scaling at surface.
Sh = Shale failing.
Su = Sutures failing.

Some modification of these symbols might prove acceptable. The list appears rather intricate. Furthermore, it would seem that the notation, "O.K.", should only be used when the piece under consideration has been unaffected throughout the test. A uniform method of reporting and a general acceptance of symbols to be used, are highly desirable. It is rather difficult to devise a set of symbols which will tell the complete story of any test. For this reason, the report blank should be accompanied by written text in which various comments upon observations during the progress of the test should be made as an aid to the interpretation of the results.

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, p. 1841.
† Numerals preceding symbols will indicate grammes of material split or chipped off.

Comments Concerning Test Procedure by Various Testing Engineers

The primary purpose of the co-operative soundness test being to check up the details of the laboratory procedure, each of the testing engineers of the co-operating laboratories was encouraged to comment specifically and make suggestions with reference to the test procedure. The results of tests by all the co-operating laboratories were submitted in complete detail to each one.

Several of the testing engineers made quite helpful comments, of which the following are typical:

Mr. J. G. Hawthorne, of the Kansas City Testing Laboratory, stated that:

"In our opinion the test as set forth in your instructions is a very satisfactory test and is reproducible by different operators. At first an examination of the results of the present series of tests would make it appear that the method is a severe method of test. This is true but it is our opinion that a severe test is necessary to insure the securing of material which will stand up in filter service over a long period of time.

"We do not believe that the test method should be refined to too great a degree for in its present form it is a simple, clear method of procedure, free from difficult manipulations, calling for simple equipment, and coming within the scope of the ordinary chemical or physical laboratory."

Based upon the results of these co-operative tests Mr. Goldbeck has suggested a sodium sulfate test procedure which he has published* subsequent to the co-operative tests. In his report to the Committee Mr. Goldbeck states that:

"It will be noted that this method differs from the outline originally described by the Committee, in that:

"1.—The sodium sulfate solution is to be saturated at 70° Fahr. instead of 86° to 95° Fahr.

"2.—Particular mention is made of the permissible range of temperature of the solution and further evaporation of the solution is prevented.

"3.—The immersed specimen must be immediately transferred to the oven.

"4.—The oven must be pre-heated to the required 100° to 105° cent.

"5.—The specimens are to be cooled before re-immersion.

"6.—The period of immersion is reduced from 20 hours to 18 or 19 hours.

"In general, the whole idea is to restrict the crystal formation to anhydrous sodium sulfate (Na_2SO_4) and not to permit any other type to form. The procedure in Paragraphs 1 and 2 insures that no crystals shall form in the solution and that in Paragraphs 3 and 4 prevents the formation of anything but anhydrous crystals during the drying period. Paragraph 5 makes for more uniform temperatures at immersion and seems to have a more severe effect on the rock. Paragraph 6 provides for a reduction in the time of immersion. This must be done because of the cooling period. However, since most of the absorption takes place within the first half hour, the change in the time of immersion has no practical effect on the test."

Dr. Kriege makes the following comment:

"The test as outlined by the Committee has emphasized several important phases sometimes overlooked in this test procedure. One of these phases is the saturation temperature of the sodium sulfate solution in any solution before crystallization can take place. Saturation with respect to that crystal must have been reached and exceeded. The point of saturation depends definitely upon temperature in the case of most salts, particularly so in the case

* *The Crushed Stone Journal*, p. 13, July, 1929.

of sodium sulfate. Obviously, therefore, the temperature at which the immersing solution of this test should be saturated is above any to which the solution will go during the soundness test if we are attempting to disrupt the stone by reason of crystals growing out of the saturated solution. If the room temperature at best may rise to 85° Fahr., the solution should be saturated at least 90° Fahr., to insure an excess of crystals present in the solution at all times.

"The examination of the samples during the test in the wet state rather than the dried state is certainly an advantage since the visibility of small cracks and disturbances is greatly reduced by a film of dried salt in the latter case. Disruption takes place during the immersion period, hence inspection should follow this immersion period.

"The fact that the temperature of the immersed solution and material is allowed to vary with room temperatures is one which may introduce variables in the results from the dozen co-operating laboratories. The temperature fluctuations within the several laboratories may be sufficiently different to cause a greater "freezing out" of the crystals in one case than in another. The ideal condition would be to cause the solution to vary a certain amount, say, 10 or more degrees Fahr., to thereby hasten the destructive action of the crystals in the immersed samples. However, such temperature control apparatus is not available in every laboratory. The next best thing is to follow the recommendation of the Committee, namely, to keep it as near room temperature as possible throughout the test period.

"Probably no comment is necessary regarding the procedure devised for the drying of these specimens. That seems to be free from objection both as to temperature and time of drying.

* * * * * * * *

"As a whole the test procedure seems to me to be concise, clear, and devoid of any technical operations, difficult to follow even in a laboratory of modest equipment."

Professor Scholer makes the suggestion that the following modified test procedure be used, and feels that such a procedure would give more uniform results and more rapid disintegration:

"Immerse the sample in a saturated solution of sodium sulfate at a temperature between 40° cent. and 50° cent. The solution to be maintained at this temperature by means of a suitable automatic control. The sample should stand at this temperature for eighteen hours and then the bath should be allowed to cool to not to exceed 30° cent. At the end of this time the pieces should be removed and examined for fracture, cracks, etc., and their condition recorded. The sample should then be placed in a drying oven for four hours at a temperature of 100° cent. to 105° cent.

"If there is any fear that the solution as it was allowed to cool to 30° cent. might be in the heptahydrate form, this could be guarded against by adding just before the samples were removed, a few crystals of the decahydrate. This would instantly turn the whole solution to the decahydrate form."

Mr. Rea states:

"I believe that while the actual freezing and thawing may give in some cases a better indication of the service behavior of the rock, it would not be practicable as a routine test. In this connection, I might say that we have been using the sodium sulfate test for about thirteen years to determine the soundness of rock for highway use and believe for the conditions we have in this State and with the kind of rock available, that the test has been of very great value. The method which we have been using is essentially the same as the method suggested by the Committee at the time of submitting the samples."

Messrs. Penniman and Browne suggested more careful temperature control and wished to protest against the adoption of a method not fully regulated as to temperature conditions.

Mr. Hubbard states:

"I have made quite a complete study of the results of the tests and have also noted carefully the suggestions made by representatives of various laboratories pertaining to the method of making the sodium sulfate test for soundness.

* * * * * * * *

"I would say that I believe the sodium sulfate test does represent a practical possibility for predetermining the comparative merits of materials for use as sewage trickling filter media.

"I think that saturating the solution at a high temperature as prescribed by your method of procedure is very good, inasmuch as it assures complete saturation at ordinary room temperatures. I believe that a closer temperature control than that obtained in ordinary room temperatures of most laboratories is not worth while, as indicated by ordinary results. The above statement is made with the assumption that the laboratory temperature is maintained fairly uniform. If conditions might exist where there was a great range in temperature in the laboratory then some more accurate control would undoubtedly be desirable."

Concluding Statement

The original selection of test samples was made with instructions to obtain a variation in character of material with respect to its durability qualities, ranging from good to poor. In the case of the limestone samples, a classification was made by Dr. Kriege and Mr. Jones of the character, based on previous experience in examining the material from the several quarries. The results of the sodium sulfate soundness tests generally agreed with the original classification of the material. In the case of the slag samples the results of the tests by the several laboratories do not indicate any appreciable difference in the character of the several samples.

Study of the results obtained by the several co-operating laboratories and of the comments made by these laboratories indicate that the sodium sulfate test is simple of execution, and gives useful results in its present form. Certain factors, particularly the matter of variations in quality of stone and in the temperature control, appear largely responsible for the differences in the results obtained. More definite temperature control can be obtained, but at the expense of simplicity of the test procedure. There is some question as to the relative merits of the additional refinement by temperature control and the simplicity of the test without these refinements. Further consideration must be given to this before a proper decision can be made as to a standard procedure to be used in the sodium sulfate test.

The results of the co-operative soundness test together with previous tests on trickling filter material indicate the possibility of the sodium sulfate test as a practicable and reliable method for determining the soundness of proposed trickling filter materials, particularly rock products. Certain modifications and the laboratory procedure may be desirable before the test can be considered to be standard.

APPENDIX II

Results of Fifty Cycles of the Sodium Sulfate Soundness Test on Samples Taken From Operating Trickling Filters with Long Records of Service

The results of twenty cycles of the sodium sulfate soundness test were reported in the 1928 Progress Report, Appendix I,* for samples of material taken from trickling filters at sixteen different cities, with a discussion of each series of tests. A summary of the results of all tests with tentative conclusions was given following the data for the individual installations.

More Extended Soundness Tests.—In order to discover the effect of continued application of the sodium sulfate soundness test treatment, the samples from the trickling filters at Baltimore, Md., Fitchburg, Mass., Gloversville, N. Y., Plainfield, N. J., and Rochester, N. Y., which have had a satisfactory service record of more than 10 years were subjected to thirty additional cycles of the soundness test, making fifty cycles in all. In some cases, some of the original material which had flaked off was lost. Furthermore, in some instances where a piece had broken into two during the first twenty cycles, only one portion of the piece was used in the subsequent thirty cycles. In the summary sheets, the weights of the individual pieces are given to the nearest gramme at the beginning of the first test, at the beginning of the second set (twenty-first cycle), and, finally, at the end. Several weeks elapsed between the end of the first twenty cycles and the beginning of the second test.

A sample of sandstone collected from the filters at Polk, Pa., was also included in this last set of tests, for thirty cycles.

Results of Fifty Cycles.—The results of subjecting the various materials to fifty cycles may be seen in Tables 8 and 9.

A study of the results obtained in fifty cycles as compared to twenty cycles indicates that the destructive force of the sodium sulfate is continuous and apparently progressive (see Table 9). All the material subjected to fifty cycles had proved satisfactory in service for a number of years, including winter operating conditions, as follows:

	Years.
Baltimore, Md.	17
Fitchburg, Mass.	11
Gloversville, N. Y.	13
Plainfield, N. J.	10
Rochester, N. Y.	12

It seems reasonable to conclude that for practical purposes twenty cycles of the sodium sulfate soundness test should be sufficient on which to predict and compare the probable service behavior of any proposed materials. It should be mentioned, that most of the breaking up that occurred in trap was clean splitting.

The results of the test on the sample of sandstone from the Polk, Pa., filters illustrate what is taking place to a less extent in the bed during actual service. Observations at the plant indicate that there is some surface abrasion going on. However, where the individual pieces are not subjected to moving and

* *Proceedings*, Am. Soc. C. E., September, 1929, Papers and Discussions, p. 1804.

TABLE 8.—Progressive Observations on Ten Samples from Each Plant, Tested for Fifty Cycles.

TABLE 8.—PROGRESSIVE OBSERVATIONS ON TEN SAMPLES FROM EACH PLANT, TESTED FOR FIFTY CYCLES.

Cycle No.	Condition of Pieces 1 to 10, Inclusive, at the End of Cycle Indicated (Numbers in Parenthesis Indicate Partial Failures in that Cycle Number).										Remarks.
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	
				Baltimore, Md.							
5	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	Pieces 5, 6, and 7 were granite; other pieces were trap-rock.
10	O K	(10)	O K	O K	O K	O K	O K	O K	O K	O K	.....
15	O K	O K O K	O K	O K	O K	O K	O K	O K	O K	O K	Piece 2 broke into two parts.
20	O K	O K O K	(20)	O K	O K	O K	O K	O K	O K	O K	.....
25	O K	O K O K	(20)	O K	(23)	O K	O K	O K	O K	O K	Piece 3 flaked.
30	(33)	O K O K	(28)	O K	(23)	O K	(27)	O K	O K	O K	Fine cracks in piece 5.
35	(33)	O K O K	(28)	O K	(23)	O K	(27)	O K	(31)	O K	Cracks in Pieces 1 and 3; Piece 7 spalled.
40	(33)	O K O K	(28)	O K	(23)	O K	O K	O K	(31)	O K	Fine cracks in Piece 9.
45	(33)	O K O K	(28)	O K	(23)	O K	O K	O K	(31)	O K	.....
50	(33)	O K O K	(28)	O K	(23)	O K	O K	O K	(31)	O K	.....
				Fitchburg, Mass.							
5	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	Pieces 3, 6, and 10 were trap-rock; other pieces were granite.
10	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	.....
15	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	.....
20	O K	O K	(20)	O K	O K	O K	O K	O K	O K	O K	Piece 3 broke into three parts and was discarded.
25	(25)	O K	....	O K	O K	O K	O K	O K	O K	O K	Bad cracks in Piece 1.
30	(30)	O K	....	O K	O K	O K	O K	O K	O K	(33)	Piece 1 spalled; Piece 10 cracked.
35	(31)	O K	....	O K	O K	O K	O K	O K	O K	(33)	Piece 1 broke into four parts.
40	(31)	O K	....	O K	O K	O K	O K	O K	O K	(33)	.....
45	(31)	O K	....	O K	O K	O K	O K	O K	O K	(33)	.....
50	(31)	O K	....	O K	O K	O K	O K	O K	O K	(33)	.....
				Gloversville, N. Y.							
5	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	All pieces were limestone.
10	O K	O K	O K	O K	O K	O K	O K	O K	O K	O K	.....
15	O K	O K	(15)	O K	O K	O K	O K	O K	O K	(13)	.....
20	(19)	O K	(20)	O K	O K	O K	O K	O K	O K	....	Piece 3 cracked; Piece 10 exploded and was discarded.
25	(19)	O K	O K (25)	(24)	O K	O K	(27)	O K	O K	....	Piece 1 cracked; Piece 3 broken into three parts.
30	(19)	O K	O K O K	(24)	O K	(28)	....	O K	O K	....	Piece 3(*c*) spalled; Piece 4 cracked; Piece 7 lost.
35	(19)	O K	O K O K	(24)	O K	(33)	....	O K	O K	....	Piece 6 spalled.
40	(19)	O K	(35) O K	(24)	O K	(33)	....	O K	O K	....	Pieces 3(*a*) and 3(*b*) cracked.
45	(19)	O K	(35) (40)	(24)	O K	(41)	....	O K	(42)	....	Piece 3(*b*) cracked; Piece 3(*e*) spalled.
50	(19)	O K	(35) (40)	(24)	O K	(43)	....	O K	O K	....	Pieces 6 and 9 spalled.
											Piece 6 cracked.

TABLE 8.—(*Continued.*)

Cycle No.	Condition of Pieces 1 to 10, Inclusive, at the End of Cycle Indicated (Numbers in Parenthesis Indicate Partial Failures in that Cycle Number).										Remarks.
	1.		2.	3.	4.	5.	6.	7.	8.	9.	10.	
	Plainfield, N. J.											All pieces were trap-rock
5	O K		O K	O K	O K	O K	O K	O K	O K	O K	O K	.....
10	(10)		(10)	O K	O K	O K	O K	O K	O K	O K	O K	Pieces 1 and 2 cracked.
15	(13)		(15)	O K	O K	O K	O K	(16)	(15)	O K	O K	Piece 1 broke into three parts; Piece 7 broke into two parts. (Piece 7(*a*) large, and Piece 7(*b*) small); Pieces 2 and 8 spalled;
20	(13)		(15)	O K	O K	(20)	(17)	(16)	(17)	O K	O K	Pieces 5, 6, and 8 cracked.
25	O K	(25) O K	(23)	O K	O K	O K	O K	O K O K	(23)	(23)	O K	Piece 1(*b*) and Piece 8 cracked; Piece 2 broken in two parts; (Piece 2(*a*) large, and Piece 2(*b*) small); Piece 9 spalled.
30	O K	(30) O K	(26) O K	O K	O K	O K	(26)	(27) O K	(29)	(27)	O K	Pieces 1(*b*) and 8 spalled; Pieces 2(*b*), 6, 7(*a*), and 9 cracked.
35	O K	O K O K	(26) O K	O K	O K	O K	(26)	(27) O K	(31)	(27)	O K	Piece 8 cracked.
40	O K	O K O K	(26) O K	(38)	O K	O K	(26)	(38) O K	(31)	(40)	O K	Piece 3 cracked; Pieces 7(*a*) and 9 spalled.
45	O K	O K O K	(26) O K	(38)	O K	O K	(26)	(41) O K	(41)	(42)	O K	Piece 9 cracked; Pieces 7(*a*) and 8 spalled.
50	O K	O K O K	(26) O K	(38)	O K	O K	(26)	(48) O K	(46)	(42)	O K	Piece 7(*a*) cracked; Piece 8 spalled.
	Rochester, N. Y.											All pieces were limestone.
5	O K		O K	O K	O K	O K	O K	O K	O K	O K	O K	.....
10	(10)		O K	O K	O K	(10)	O K	O K	(10)	O K	O K	.....
15	(15)		(15)	(15)	O K	(15)	(15)	O K	(10)	O K	O K	Piece 1 cracked; Piece 5 spalled; Piece 8 broke into two parts.
20	....		(15)	(20)	O K	(15)	(20)	O K	(10)	O K	O K	Piece 7 exploded to fines; Pieces 2 and 3 cracked; Piece 5 sandy; Piece 6 spalled.
25	....		(15)	(23)	O K	(23)	(23)	O K	O K	O K	O K	Piece 3 cracked; Piece 6 sandy.
30	....		(15)	O K	O K	(23)	(23)	(29)	(26)	O K	O K	Pieces 3 and 5 spalled; Piece 6 broke into five parts.
35	....		(15)	O K	O K	(23)	(23)	(33)	(34)	(32)	O K	Piece 7 spalled; Piece 8 cracked.
40	....		(15)	O K	O K	(23)	(23)	(33)	(38)	(32)	O K	Pieces 7 and 9 cracked; Piece 8 spalled.
45	....		(15)	O K	O K	(23)	(23)	(33)	(38)	(32)	O K	Piece 8 broke into three parts.
50	....		(15)	(46)	O K	(23)	(23)	(33)	(38)	(32)	O K	Piece 3 cracked.
	Polk, Pa.											All pieces were sandstone.
5	(4)		O K	(2)	(4)	O K	O K	(4)	O K	(2)	(4)	Piece 1 crumbly; Pieces 3, 4, 7, 9, and 10, sandy.
10	(4)		(10)	(2)	(4)	(6)	O K	(4)	(6)	(6)	(4)	Pieces 5 and 9, crumbly; Piece 8, sandy; Piece 2 spalled.
15	(12)		(15)	(2)	(4)	(6)	O K	(4)	(6)	(6)	(4)	Piece 1 cracked; Piece 2 spalled.
20	(12)		(18)	(2)	(4)	(6)	(17)	(4)	(18)	(6)	(4)	Pieces 2 and 8 broke into two parts; Piece 6 sandy.
25	(23)		(25)	(2)	(4)	(6)	(21)	(25)	(21)	(6)	(25)	Pieces 1, 6, 7, and 10, crumbly; Piece 2 cracked; Piece 8 broke into two parts.
30	(26)		(26)	(2)	(4)	(6)	(21)	(25)	(26)	(6)	(25)	Pieces 1 and 2 broke into three parts; Piece 8 cracked.

TABLE 9.—WEIGHT OBSERVATIONS ON TEN SAMPLES FROM EACH PLANT DURING FIFTY CYCLES OF TEST. (SEE TABLE 8.)

Cycle No.	Weight (to the nearest gramme) of pieces 1 to 10, inclusive, at the end of cycle indicated.										Total.	Per-centage lost.
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.		
BALTIMORE, MD.												
0	105	174	239	155	209	213	156	279	152	347	2 029	
20*	106	175	233	155	209	213	155	278	151	346		
Loss†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	19	0.94
50*	105	174	233	154	208	212	153	278	151	346		
Total loss†	0	0	6	1	1	1	3	1	1	1	15	0.74
FITCHBURG, MASS.												
0	131	67	81	56	90	109	67	159	107	97	964	
20*	131	67	73	56	90	109	66	159	106	97		
Loss†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	10	1.04
50*	94	67	71	56	89	109	66	159	106	97		
Total loss†	37	0	10	0	1	0	1	0	1	0	50	5.2
GLOVERSVILLE, N. Y.												
0	84	230	200	82	51	107	44	58	51	92	863	
20*	83	230	199	81	51	107	.....	58	51	.....		
Loss†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18	1.8
50*	81	229	169	79	51	91	.....	58	43	.....		
Total loss†	3	1	31	3	0	16	.....	0	8	.....	62	7.2
PLAINFIELD, N. J.												
0	183	160	152	175	227	175	257	156	167	93	1 745	
20*	167	150	148	174	226	174	256	129	165	92		
Loss†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	19	1.1
50*	162	148	146	173	225	173	245	119	162	92		
Total loss†	21	12	6	2	2	2	12	37	5	1	100	5.7
ROCHESTER, N. Y.												
0	155	98	85	126	200	102	207	133	96	95	1 297	
20*	46	97	82	125	194	97	206	105	95	94		
Loss†	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	152	11.7
50*	0	97	72	125	179	60	204	64	95	94		
Total loss†	155	1	13	1	21	42	3	69	1	1	307	23.6
POLK, PA.												
0	375	332	171	186	233	232	241	176	276	187	2 409	
30*	139	287	159	174	226	212	216	82	169	167		
Total loss†	36	45	12	12	7	20	25	94	107	20	378	15.7

* Pieces larger than 1 in.
† Pieces smaller than 1 in.

rubbing, as they are during a test, or any other handling, the abrasion loss would be less. This material has given satisfactory service for nineteen years. It was placed originally because no other suitable material could be obtained economically. Test on the large sample showed only 5.5% absorption. The greatest objection to sandstone apparently lies in the difficulty of placing it without undue breaking up during handling. Viewed from a theoretical standpoint it should furnish excellent filtration results as the interstitial water between doses apparently would be quite large since the entire piece appears to be saturated when wetted.

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PAPERS AND DISCUSSIONS

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FILTERING MATERIALS FOR WATER AND SEWAGE WORKS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION

Discussion*

By Messrs. James W. Armstrong and W. H. Dittoe

James W. Armstrong,† M. Am. Soc. C. E.—The work of the Committee in connection with sand for mechanical filters has been carried on through the various co-operating cities. Fourteen different cities agreed to conduct these experiments, and a few of them have done some very excellent work; but for one reason or another the progress has been slower than the Committee hoped it would be. A number failed to carry on because they did not have sufficient laboratory help; others, due to mechanical troubles which took their entire time, could not continue the experiments.

Thus far, the Committee has confined its efforts to an endeavor to determine the best size and depth of filter sand. In order to draw any general conclusions that would be of value throughout the United States, it was deemed necessary to carry on the experiments simultaneously in different parts of the country with different kinds of water. To reduce the variables and the personal equation to a minimum, all the sand used was screened through the same set of sieves and was sent to the different co-operating cities ready for placing in the various filter units.

The Committee does not feel justified in drawing any definite general conclusions at this time regarding the best size and depth of filter sand; but by plotting and comparing the results of a great many different filter runs, a marked similarity was observed in the performance of the filters equipped with the same sand, whether it applied to different cities or to seasonal variations

* This discussion (of the Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works, presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1930, and published in this number of *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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in the same city. There was no such thing as uniformity in the length of filter runs, but the variations in length of runs of the small units were no greater than the same run on the large-sized filter units.

Indications are that for turbid or moderately turbid water, or for water with high organic content, the sand sizes generally accepted as best suited to mechanical filters are too fine. From a preliminary study of the information in hand, fine sand gives shorter filter runs without apparently improving the quality of the effluent. There seems to be a decided advantage in the use of coarse sand. For instance, a filter containing sand in the top layer of 0.4 mm., or less, in diameter, gives runs too short for the best operating conditions. Better results seem to be secured from filters in which the top sand layer is from 0.5 to 0.62 mm. in diameter.

The term, "top layer", is preferable to "effective size", because the latter is misleading. When floc-ladened water, in passing through a filter, strikes the sand layer, the floc immediately breaks; hence, in a mechanical filter the effective size, if the term can be used without confusing it with its generally accepted meaning, is the extreme top of the sand bed. The term, "effective size", is graphic and appropriate, but it has a meaning too well fixed in engineering literature to attempt to use the expression and give it a new meaning. Some other expression such as "top size" should be adopted to indicate the size that performs the effective work of a mechanical filter. As far as depth of sand is concerned, there seems to be little difference in the results obtained from filters in which the total depth of sand varies from 1.7 to 2.5 ft. Clear soft water free from organic matter can probably be filtered satisfactorily with finer sand than waters of the opposite kind.

W. H. Dittoe,* M. Am. Soc. C. E.—The field of investigation which might be attempted by the Committee is of wide scope and embraces a number of important questions such as loadings, rates, sizes, efficiencies, etc., all related more or less directly to the subject of filtering material in water and of sewage treatment practice. If all these various matters are to be studied it is apparent that the Committee cannot complete its work for many years.

The outstanding question which brought about the original appointment of this Committee was that of durability of filtering materials for trickling filters in sewage treatment practice. This question has received the principal attention of the Committee and perhaps could be reported upon fully during the coming year if the report is not to be withheld for study of matters concerning relative suitability of materials and effects of loadings and other factors.

There is much to be said in favor of the submission of a report dealing in a more or less final manner with the question of durability without awaiting the further study of other questions. In determining what other studies should be undertaken in regard to materials for both sewage and water treatment processes the Committee should be guided by the desires of the Division, and it is to be hoped that the members of the Division will give the Committee the benefit of their opinions as to further investigational work.

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SPILLWAY DISCHARGE CAPACITY OF WILSON DAM

Discussion*

By Louis G. Puls, Assoc. M. Am. Soc. C. E.†

Louis G. Puls,‡ Assoc. M. Am. Soc. C. E. (by letter).§—It becomes necessary to call attention to an algebraic error in Equation (2),‖ which, when correctly stated, is,

$$\frac{x^2}{b^2} + \frac{y^2}{A^2} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

It is the corrected integral equation that Mr. Pearce has in mind when he states:¶ "Equation (7) is in a form which makes the arithmetical work laborious and a simpler equation would be desirable."

Acknowledgment is here made to Mr. Pearce for calling attention to this error. It is necessary to revise the part of the paper affected by it. Consequently, the writer chooses to substitute the parabola equation, and Table 5 is re-computed on that basis. The part of the original paper that is here revised is all (except the last two paragraphs) of the section entitled, "Discharge Formula with Gate Partly Opened."** This revised chapter should read as follows:

To determine discharge it is necessary to express the vertical velocity area as a function of the length of the spillway and to integrate between the limits of this length. Since it varies increasingly from the pier to the center of the gate-opening according to some function most probably of the second degree, the vertical velocity area can be expressed as a function of this distance by the equation of the parabola. No doubt there is a rapid decrease in the rate of

* Discussion of the paper by Louis G. Puls, Assoc. M. Am. Soc. C. E., continued from February, 1930, *Proceedings*.

† Author's closure.

‡ With Hugh L. Cooper & Co., Inc., Kitchkas, Ukraine, Union of Socialistic Soviet Republics.

§ Received by the Secretary, April 23, 1930.

‖ *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2137.

¶ *Loc. cit.*, January, 1930, Papers and Discussions, p. 159.

** *Loc. cit.*, October, 1929, Papers and Discussions, p. 2136.

change of this area within a few inches from the masonry pier and, consequently, the curve departs from any definite function of this spillway length. However, the percentage of error is small and can be neglected. The equation of the parabola:

$$X^2 = 4p\ (y - A) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

is established by solving for the constant, p, in terms of A, B, and e (see Fig. 6*). The vertical velocity area, as found from Fig. 3,† is here designated as the ordinate, A, and in moving according to Equation (15) between the limits of $x = 0$ and $x = \frac{L}{2}$, it will generate a volume representing one-half the discharge.

The derived equation for the total discharge of one spillway will be:

$$Q = A\ L - \frac{(A - B)}{12\ e^2}\ L^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In the special problem under discussion, e and L are constant; that is, $L = 38$ ft. and $e = \frac{38}{2} - 3 = 16$.

TABLE 5.—Computed Discharge with Adjacent Gates Closed.

Gate opening, in feet.	A.	B.	Q, in cubic feet per second.
1	19.2	19.0	726
2	39.3	38.2	1 474
3	58.6	57.2	2 202
4	78.3	77.0	2 952
5	98.2	96.0	3 692
6	117.6	114.0	4 404
7	136.2	130.6	5 076
8	153.5	145.6	5 692
9	170.8	160.6	6 308
10	188.6	178.0	6 978
11	205.5	197.0	7 657
12	224.0	216.5	8 378
13	243.6	236.5	9 130
Full gate	281.6	245.9	10 063
(Corrected value)	.....	.....	9 925

Computation data for the discharge through each foot of gate-opening are arranged as shown in Table 5. The values of A and B are taken from Fig. 3. It will be noted that Table 5 gives the discharge for full gate-opening to be 10 063 cu. ft. per sec., as computed by Equation (16). For this particular opening the parabolic curve integration may give a result greater than the actual, due to the greater difference in the velocity areas, A and B. If the velocity area at A is divided by the depth of the water at that point (see Fig. 5‡), the average velocity for the section is $\frac{281.6}{14.97} = 18.8$ ft. per sec. Similarly, the average velocity for the section, B, is $\frac{245.9}{13.15} = 18.7$ ft. per sec.

* *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2137.
† *Loc. cit.*, p. 2135.
‡ *Loc. cit.*, p. 2136.

This shows the average to be nearly equal in the two sections. The total area of the discharging jet at full gate is 529.8 sq. ft. (see Fig. 5). This will give a discharge of $529.8 \times 18.75 = 9\,925$ cu. ft. per sec.

Inspecting the discharge quantities shown in Table 5, it will be readily noted that the average line plotted through these points will not vary appreciably from the curves as shown in Fig. 7.* Consequently, the writer has not considered it necessary to change this graph.

In regard to Mr. Pearce's discussion of the discharge with adjacent gates open, the writer wishes to emphasize that no effort was made in the paper to express a general law for the increment of discharge due to the influence of motion of the neighboring water mass. Referring to Fig. 2† it will be noted that two velocity measurements were made with adjacent gates fully opened and these measurements indicated that there was an increase of velocity as compared to the conditions with the adjacent gates closed. It should be stated that the increase is due to end contractions or any other influences. In passing it may be well to advance the theory that the elimination of friction between the water trains approaching the crest has a greater influence than the change of contraction between the two piers. No attempt was made in the paper, however, to invade the opinions or laws advocated by older authorities. The writer measured the increment of velocity at the depth stated and it is perfectly permissible to calculate the increment of head that would cause this velocity. Furthermore, it is logical to apply it as effective over the entire discharge area because, if any increment of head will increase the velocity at a point 2 ft. above the crest, it may also be expected to increase the velocity at higher elevations truly according to the fundamental law of falling bodies.

* *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2140.
† *Loc. cit.*, p. 2134.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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LAMINATED ARCH DAMS WITH FORKED ABUTMENTS

Discussion*

By A. A. Eremin, Assoc. M. Am. Soc. C. E.

A. A. Eremin,† Assoc. M. Am. Soc. C. E. (by letter).‡—The author has indicated a number of limitations of the present methods used in designing thick arch dams due to the "uncertain assumptions" on which computations of the unit stresses are based. In order to simplify doubtful complicated mathematical manipulations in design of arch dams the author introduces a new type of laminated arch dam composed of thin arches which are separated from each other by sliding joints or films of water. Since the unit stresses in sections of thin arches are more uniform than in thick arches the error in determining stresses in the laminated arch dams by the methods based on the cylindrical formula will be less.

The idea of subdividing compressed thick arch dams is not new; it was proposed in 1894 by the French engineer, Boulé; in 1912 by the Italian engineer, Rutenberg; and, recently, it was applied to actual practice in the French dam on the Dordogne River as stated by Mr. Noetzli.§

The horizontal deflections of thick arch dams are generally small and their effect on the primary stresses in an arch is commonly neglected. However, in thin arches, deflections are greater and their influence on the stability of the arch must be considered.

For a circular arch with hinged ends acted upon by a uniform radial pressure Professor S. Timoshenko developed the following expression for critical pressure or the maximum pressure beyond which a structure might buckle:‖

$$q_{cr} = \frac{E\,I}{R^3}\left(\frac{4\,\pi^2}{4\,\phi^2} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

* Discussion of the paper by Fred A. Noetzli, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Sacramento, Calif.

‡ Received by the Secretary, July 24, 1930.

§ *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 284.

‖ "Applied Elasticity," by S. Timoshenko and J. M. Lessels, First Edition, 1925, p. 247.

It is evident from this formula that the buckling effect in thin arches is inversely proportional to the ratio, $\frac{R^3}{I}$, or $\frac{R}{t}$. Furthermore, in concrete arch dams with central angles of less than 90°, the buckling of the structure need not be feared.

The author has stated* that, neglecting the effect of cantilever action, "the deflections of corresponding points along radial lines will be the same for all arch laminæ." Within practical limits, this statement is correct when it refers to the upper parts of thin arches that are fixed at the bottom. However, near the bottom, deflections vary along the axis of the arch, thus inducing a variation in the distribution of the external pressures carried by arches unless a film of water is provided between laminæ.

Professor C. Guidi made an experimental test to determine the deflection of a circular concrete arch 16 ft. 4¾ in. high, 27 ft. 10½ in. long, and 6¼ in. thick, with a central angle equal to 180 degrees.† This arch was fixed at the bottom and both ends, and was subjected to a uniform radial pressure. In Fig. 18 the curve, *AC*, represents the deflection of this arch at Elevation 5 ft. 9 in. Its original unloaded position is represented by the curve, *AB*.

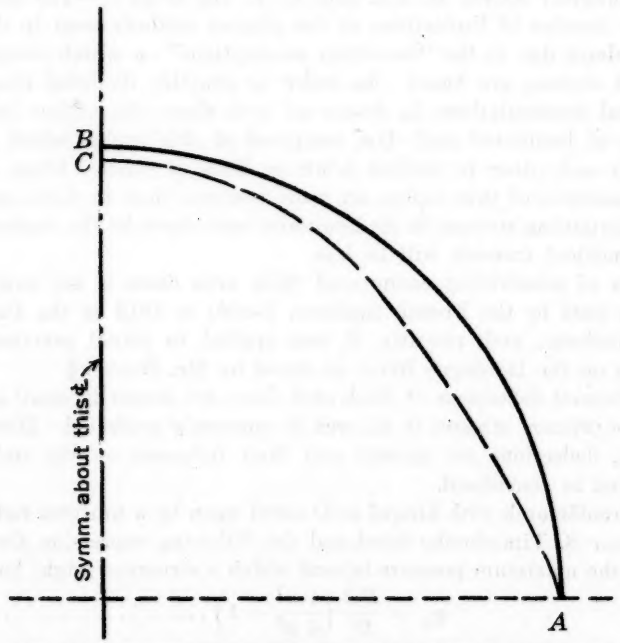

FIG. 18.—RELATIVE DEFLECTION OF AN ARCH DAM SUBJECTED TO A UNIFORM RADIAL PRESSURE.

The effect of the variation in deflection of thin arches in the laminated arch dam on primary stresses is probably small. However, it might be rapidly increased with an increase in the number of laminæ used in the dam.

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 277.
† *Revue Générale de l'Electricité*, October 29, 1927.

The advantages of using forked abutments have been well indicated by the author.* However, when there are certain topographical conditions at a dam site, such that a high, gravity type of wing-wall would be required, the French buttresses with arched flanks, as shown in Fig. 9,† would be more suitable.

The arched flanks are more economical than the high, gravity type of wing-walls and the stresses in the buttresses are symmetrically distributed about the center line of the arch. Therefore, the buttresses may be of plain concrete. Steel reinforcement in the main structural members of dams is frequently objectionable because of possible corrosion.

By inclining the center lines of buttresses toward the center line of the main arch in the French type of laminated dam efficient security against sliding is obtained.

The Engineering Profession should appreciate highly the author's development of this theory of designing arch dams and his bold application of the scientific principles to actual practice.

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 275.
† *Loc. cit.*, p. 285.

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GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

PREPARED BY CONFERENCE COMMITTEES FROM THE AMERICAN SOCIETY OF CIVIL ENGINEERS AND THE AMERICAN RAILWAY ENGINEERING ASSOCIATION

Discussion*

BY MESSRS. C. A. P. TURNER, AND GUNNI JEPPESON.

C. A. P. TURNER,† M. AM. SOC. C. E. (by letter).‡—The recommendation of pin-connected trusses for spans of 150 ft., or longer, in Article 102§ of the specifications, is difficult to harmonize with the fact that the riveted truss up to at least 500 ft. in span is more rigid, permits better bottom lateral connections, simplifies floor-beam details (uncut for pins), weighs less, costs less for fabrication, and can be erected more readily by adopting fewer bottom chord joints. A stiff bottom chord throughout further permits partial cantilever erection at times effecting a large saving under difficult conditions.

Equally inexplicable is the suggestion in Article 104,§ of adopting one-tenth the span length as a satisfactory depth of truss when, for economy and stiffness in the through truss, the ratio should never be less than one-fourth the span up to 200 ft., one-fifth, for 300 ft., and one-sixth, for 500 ft. Two-thirds of these latter ratios, however, are unobjectionable for continuous deck structures.

While the wording of the specifications is advisory rather than mandatory, adoption without question by younger members would lead to inferior designs, with 40 to 50% increase in weight and threefold or fourfold greater deflection than would accrue with economic depth ratios.

Article 300.‖—Progress in physical science is a slow, step-by-step process, building upon continued experiment and practical experience. Disregarding this principle the Conference Committees advocate raising the old working

* Discussion of the General Specifications for Steel Railway Bridges, continued from September, 1930, *Proceedings*.

† Cons. Engr., Minneapolis, Minn.

‡ Received by the Secretary, July 8, 1930.

§ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2650.

‖ *Loc. cit.*, p. 2653.

stress* of 15 000 lb. per sq. in. for soft steel to 24 000 lb. per sq. in. for dead loads. No new experimental evidence is brought forward to substantiate such a radical departure from established practice, and no more cogent argument appears than the unwarranted presumption that the elastic range of stress may be substantially measured by the yield point of the metal. As the latter, without noticeable plastic deformation, can readily be raised almost to the ultimate strength by repeated application of load below, but approaching, its value, the dangerously speculative character of the presumption is apparent.

In thus raising the primitive yield-point value, residual strain is accumulated and work is performed on the structure of the metal tending to wear it out.† The lower limit of this phenomenon is about five-eighths of the primitive yield point. Hence, working stresses of 24 000 lb. per sq. in. for soft steel lie beyond the range of true elasticity, and structures so designed cannot be regarded as permanent. Conversely, a range of stress—whether it is all compression, all tension, or varying from tension to compression—within one-half the primitive yield point tends to improve the elasticity, ductility, and tenacity of the metal. This is brought about by the dissipation of residual strain and the consequent improvement in the elastic coefficients.

Article 301.‡—The shear and bearing values for rivets in the specification are too high. A review of the irrational rise in unit shear and bearing values during the past forty years substantiates this viewpoint.

In 1890, the customary value for shear on rivets of railroad bridges was 6 000 lb. per sq. in., and for bearing, 12 000 lb. per sq. in. As steel came into use this was raised to 6 600§ or 7 500 lb. for shear and 15 000 lb. for bearing, while, in building work, 9 000 lb. per sq. in. in shear and 18 000 lb. in bearing seem to have been the highest values considered within reason. The specification of 1908, Department of Railways and Canals of Canada, permitted 10 000 lb. in shear and 20 000 lb. in bearing, but the material was to be medium steel. The Pencoyd Handbook of 1898 gives only three tables for rivets—shear at 6 000, bearing at 12 000; shear at 7 500, bearing at 15 000; and single shear at 11 000 and bearing at 22 000. The higher value was used in connection with an impact allowance of 80 or 100% increase in the live stress on the bridge member.

Since 1890, shear values have increased from 6 000 lb. per sq. in. for iron rivets and from 6 600§ to 7 500 lb. for mild steel rivets to 24 000 lb.‖ per sq. in. in the Conference Committees' specification. Obviously, if these values keep on increasing something serious will happen.

The resistance of a rivet for lower intensities of stress is predominantly a resistance to sliding of the parts held together by the tension on the rivet which is caused by shrinkage. A tight rivet which has cooled from 800 to 900° Fahr., ordinarily shrinks sufficiently to develop 15 000 to 18 000 lb. per sq. in. in ten-

* "Specifications for Railway Bridges," by the late C. C. Schneider, Past-President Am. Soc. C. E., Pencoyd Handbook, 1898.

† "Thermo-Electric Measurement of Stress," by C. A. P. Turner, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), p. 140.

‡ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2654.

§ "Specifications," by F. H. Lewis, *Proceedings*, Engrs. Club of Philadelphia, 1891; also, Kent's Mechanical Engineers' Pocket Book, 1903, p. 380.

‖ Now revised to 12 000 lb. and 13 500 lb. for buildings in the specifications of the American Steel Institute, to aid in meeting the competition of concrete.

sion, gripping the parts together if they have been well bolted in assembly. If the coefficient of static friction is taken at 0.4 at the initial slip, there develops a nominal shear of 8 000 or 9 000 lb. per sq. in. The elastic stress to which the rivet is then subjected would consist of the resultant of the shrinkage tension at the slip, combined with the elastic tensile stress from bending. The elastic stress in shear, which is computed for the combination of these states of stress by the methods taught in books on mechanics, may be nearly four times the magnitude of the unit shear stress computed on the cross-section of the rivet. It follows then that the elastic resistance of the rivet has been exceeded, and permanent set must take place under working conditions.

With the strenuous effort now made to keep working stresses (and even minor secondary stresses) within the limits of elasticity of the material of which great bridges are constructed, such major stresses as are readily approximated in the rivet by the elastic theory should receive careful consideration.

In a web member connection that had been in service fifteen years, all sixteen rivets of the joint were found to be loose, working and wearing like "fitting-up" bolts. Had these rivets been designed for the 24 000 lb. per sq. in. in shear of the Committees' specification, instead of 8 000 lb. per sq. in., the structure would not be standing to-day. Even with the revision to 12 000 lb., its stability would be open to question.

Tests of large steel columns presented by the late Mr. J. E. Howard,* disclose yielding of pin plates under values but slightly in excess of the bearing value of the Committees' specification for turned bolts. (See Article 301.)

The average initial slip value for carbon-steel joints (Canadian Government report† on the Quebec Bridge), occurred when the nominal shear on the rivet was a little more than 9 000 lb. per sq. in., yet the Committees' specification permits a revised shear value of 12 000 lb. per sq. in. Added to the uncertainty of providing for the maximum stress on individual rivets is the assumption of the uniform average resistance of such rivets regardless of the true elastic distribution of stress throughout the varied formation of groups of rivets that comprise a joint.

Again, the high rivet replacement record of the old Victoria (tubular) Bridge designed under low nominal shear values brings into relief the unwarranted confusion of external shear force with internal shear strain in restrained and continuous plate-girder design.

The hypothetical negation of Newton's third law in textbook theory of flexure is responsible for this confusion and erroneous rivet spacing closest at the support instead of at the point of contraflexure.

The Pottsville Iron and Steel Company used medium in lieu of soft steel for rivets in 1896. The tested values were from 15 to 25% higher than for soft steel and no difficulty was experienced in driving them satisfactorily in the shop. Nickel-steel rivets in test records for the Quebec design disclose somewhat higher ratios of improvement in ultimate strength. From this record of test and experience in use a stronger and tougher metal is desirable for rivet stock. A medium steel low in sulfur and phosphorus with its hot ductility

* *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), pp. 442–443.

† Quebec Bridge Rept., Vol. I, p. 188.

improved by a small quantity of chrome, would add little to the cost and largely to the strength of rivet steel for bridge construction.

Rationally balanced working stresses in a framed structure should be selected so that compression and tension members have equal capacity to withstand overload without injury. In the tension member the fiber stresses within the yield point increase substantially in proportion to the load, but in the compression member the increase takes place at a far more rapid ratio.

The arbitrary rules-of-thumb for slenderness ratios in column design of the past decade give no indication of the law of this increase, but the brilliant investigation of William Crehore in 1879, combining the Rankine and Euler column formulas, makes it possible to take this variation into consideration by the formula,

$$u = \frac{P}{A} = \frac{f}{1 + \frac{n f L^2}{\pi^2 E r^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which, P is the applied load; A, the area of the member; L, its length, in inches; r, the radius of gyration; n, a coefficient dependent upon the end restraint; f, the extreme fiber stress, in pounds per square inch; and E, Young's modulus.

To correlate the working stresses in compression and tension the mean working stress in design should be one-half the mean compression, u, which will produce a maximum fiber stress of twice the working value assumed for tension. Then, if the tension member can sustain double its working load within the yield point of the material, the compression member thus designed may likewise do so without acquiring a permanent set.

When the construction is riveted and the ends of the compression member fixed, $n = 3$; when the joint is a pin-bearing, with a diameter of pin of not less than one-third the breadth of the member, a value of $n = 1\frac{1}{2}$, in lieu of $n = 1$, may be used for knife-edge bearings.

The Conference Committees' formula for column design (Article 301(*a*)*) may be weighed by considering the relative strength developed by pipe, angle, and T-struts of identical material, area of section, and slenderness ratio, but differing in make-up and end restraint.

When $\frac{l}{r} = 140$, the pipe strut with flat ends is approximately two and one-half times as strong as the angle-iron strut on ball and socket bearings, and the pipe strut either round or pin-ended, is twice as strong as the angle iron with ball and socket bearing. The angle or T-strut, pin ended, is one and three-quarters times as strong as with round ends. Nevertheless, under the Committees' formula the computed strength would be identically the same for each.

The pipe strut, being symmetrical, has no tendency to twist because that would increase its length. The angle or T by twisting relieves the outer fibers of part of the load, and the inner fibers are called upon to carry increased stress.

The striking similarity of the Queensborough Bridge (New York, N. Y.) sections to those of the first Quebec design brought its safety into question.

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2654.

The fundamental difference lies in diaphragms,* spaced on 8-ft. 5-in. centers, in the Queensborough structure, and a greatly reduced slenderness ratio of the chord, thereby reducing the relative twisting weakness of the member to approximately 80% less than that of the Quebec chord and hence it is reasonably safe. Nevertheless, if its section were used in a longer and more economic panel this would not be true.

The Crehore formula (Equation 21), like those of Rankine and Euler from which it was derived, considers only pure bending and end restraint of a solid or symmetrical section, such as a hollow cylinder or pipe. Because in providing for bending stiffness the working stresses for a column decrease rapidly as the slenderness ratio increases, various unsymmetrical built-up sections have been devised to meet this difficulty. A unit strip at the extremity of an angle, an **H**, or a **Z**-bar section is relatively lacking in lateral rigidity; it tends to buckle out of line normal to the direction which the body of the strut bends under axial compression, thus giving rise to the twisting or buckling action exhibited by **Z**-bar columns tested for the American Railway Engineering Association by the U. S. Bureau of Standards. The middle part of the thinner sections rotated around the column axis, while the ends held by the machine heads stayed in their original position.† Such torsional action of the column section unsymmetrical about its central point was responsible for the failure in June, 1891, of the Continental Railroad Bridge over the River Birs, near Mönchenstein, Switzerland, and others similar in make up, and the disastrous collapse of the first Quebec design may be traced to the same cause.‡ Provision for additional bending due to secondary stress has accomplished nothing toward rectifying and improving this kind of torsional weakness of unsymmetrical sections in common use to-day. No comprehensive specification has come into common use for the elimination of such weakness.

While wide divergence of ultimate strength decreases with decrease in the ratio of $\frac{l}{r}$, the elastic range of stress differs to a more marked degree than the ultimate strength.

For a steel angle column made up of 4 ft. 6 in. by 3½ in. by ⅜-in. angles, 11 in. back to back, 25 ft. 0 in. in length from center to center of pins, with one 5 1/16-in. pin at one end and a 4 11/16-in. pin at the other, Mr. C. P. Buchanan§ found an elastic limit of 12 600 lb. per sq. in., with an ultimate of 27 700 lb. per sq. in. The slenderness ratio was 97, the material was steel of 62 000 to 65 000 lb. per sq. in. ultimate strength, and a yield point of 39 000 lb. per sq. in.

Comparing this with the Phœnix iron column sections tested at the Watertown Arsenal, having a slenderness ratio of 100, the elastic limit developed was 27 960, and the ultimate strength 35 270 lb. per sq. in. This difference is far too great to attribute to the difference between pin and square ends, to say nothing of the fact that the angle-iron section was of medium steel and the Phœnix section of wrought iron. In this instance the combination of craftsmanship with wrought iron of the Phœnix section doubled the effective yield point compared to that of the steel column.

* Royal Comm., Quebec Bridge Inquiry, Drawing 35.

† "Elasticity and Strength of Materials," Section IV, by Turner, p. 54.

‡ *Engineering News*, August 1, 1891, pp. 92-93; and *Engineering News-Record*, June 28, 1917, p. 641.

§ *Engineering News*, December 26, 1907, Vol. 58, No. 26, p. 685.

Tests* on scale models of large present-day compression members with the favorable slenderness ratio of 25 to 40 developed a low effective elastic limit in nickel, chrome, silicon, and hard steel specimens. The yield point of the metal was higher on the average than the ultimate tensile strength of the wrought iron in the Phœnix section. The slenderness ratio of the Phœnix section was three to four times that of the scale model; yet it developed a higher average effective elastic limit.

The misleading similarity of the chords of the Queensborough Bridge, the Thebes Bridge, and the Monongahela Bridge to the sections of the first Quebec design is strikingly illustrated in Drawings 34, 35, and 36 of the Royal Commission's report. The torsional weakness of such sections should be guarded against: First, by the use of solid instead of laminated webs; second, by concentrating not less than one-half the section in the flange; third, by developing flange stiffness by specifying that the width of the flanges be not less than one-fourth the depth of the web; and, fourth, by the use of substantial single lacing of ship channels or angles possessing sufficient stiffness to add torsional rigidity to the member. Knick-knack latticing of diminutive flats or small angles attached in a delicate manner to a narrow flange with one or two isolated stitch rivets is not conducive to the development of the highest resistance of the section.

Through lack of craftsmanship, present-day practice appears to be developing not more than three-quarters of the ultimate strength, nor more than five-eighths of the effective yield-point values in large built columns of which the metal is capable. When a chord section requires 250 sq. in., or more, an octagon form avoids the twisting tendencies of the unsymmetrical section. Side-plates may be secured in long lengths, avoiding the numerous butt joints of the Forth Bridge column type. The corner joints may be economically welded, backed by a 3 by 8-in. beveled corner bar and stayed by ring frames at 10-ft. intervals. Such a section will develop the advantages claimed at a large reduction in cost of fabrication.

Shop Work.—The Committees fail to include a specification for welding which will soon find an important place in bridge-shop practice.

Article 204.†—The Conference Committees have failed to study impact from the standpoint of the craftsman. When a concrete centering shore, carrying heavy weight, is struck with a maul, it is relatively rigid and it springs only a little under the blow as contrasted to its readiness to bend when the load has been decreased by the drying and curing of the concrete work. In like manner the bridge member under dead stress is much less affected by impact or sudden loading. The writer,‡ Gustav Lindenthal, Hon. M. Am. Soc. C. E.,§ and others have applied this elementary fact in derivation of more rational impact allowances than those proposed by the Conference Committees.

Article 444.‖—Observation of roller expansion joints in service indicates that they function most efficiently as a joggle joint when new, and prior to becoming clogged with dust. After they are somewhat gummed with dirt or

* *Technological Paper No. 101,* U. S. Bureau of Standards.
† *Proceedings,* Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2652.
‡ See "Elasticity and Strength of Materials," Sections I and V, by Turner.
§ *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 922.
‖ *Proceedings,* Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2661.

dust they become usefully efficient in dampening impact vibrations; but about the time they become really efficient in this respect they seem to be less satisfactory for their primary purpose.

Considering the fact that the Forth Bridge, with a span of 1 600 ft., has survived for the past forty years on a plane, lubricated, sliding shoe-plate, the mandate of the specification that an ordinary railroad bridge span shall be provided with roller bearings appears irrational and wasteful. This is more particularly true since the advent of the grease gun which permits the usual scores in the upper sliding surface to be solidly filled with hard grease every three or four years when painting the bridge.

Such a bearing possesses the advantage of lowest possible cost, is absolutely dust-proof, properly dampens impact vibration, and permits the same detail to suffice for fixed and expansion ends in masonry and bed-plates anchors and shoes, except for grease scores.

*Article 442.**—To secure the highest rigidity no joggle joints in floors of fixed spans should be permitted, regardless of length.

Article 203.†—Increase in engine-wheel concentrations from 40 000 to 64 000 lb. for computation, although pictorially impressive as a recognition of the increase in weight of present-day rolling stock, is meaningless from the practical standpoint of increased safe strength when unit tensile bearing and shear stresses are irrationally increased far above the limits of older and more conservative specifications. Because the effect of impact is accentuated at the joints of a bridge truss, increase of shear values, due to an impact allowance, is objectionable.

In the "old days", every time the Mechanical Department of a railroad company bought or made a new locomotive the exact weight and spacing of the wheels were incorporated in a new bridge specification. Instead of "tinkering" with the spacing of wheels on a wheel-load diagram one would have expected from the Committees a useful discussion of rational variation in loading for different lengths of span and segments of span. Every wheel-load diagram can be reduced to an equivalent uniform load curve for the various spans and segments of spans to which it is to be applied in practical computation. When this is done a simple unit shear diagram renders the computation of stresses practically as simple as if the load were uniform. If such equivalent loads, for the segments‡ loaded, are considered as panel loads in the same way that uniform load is usually considered, the web stresses will exceed by a small amount those computed from the exact moment diagram. Inasmuch as this excess is small and is on the side of safety for practical purposes of computation, the desired results are secured in 2 or 3% of the time that would be involved in needlessly tedious computation from wheel loading.

A plot§ of the curve of variable equivalent loading will disclose the characteristics of variation and the manner in which such variation may be modified to meet all requirements. Because the Cooper *E*-loading has been customarily used for years it is firmly established as a measure of the strength of bridge structures in the minds of engineers, bridge builders, and railroad men. Any

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2661.
† *Loc. cit.*, p. 2651.
‡ "Elasticity and Strength of Materials," Section V, pp. 4–5.
§ *Loc cit.*, p. 8.

variation from this should be made only as new engine loadings demand attention, because of the lack of regularity in variation of the equivalent load it represents.

For years, the locomotive suitable singly for the train on a low grade has been coupled to another locomotive and operated as a "double header" on steep grades. The present trend of practice in mountain divisions is toward the Mallet type, differing from the typical "double header" in that the heavy concentrations are crowded closely together instead of being spaced the length of a locomotive apart.

Turning now to the equivalent uniform load diagram for the Cooper *E*-60* it is found that for spans varying from 20 to 150 ft., there is an irregular downward kink in the equivalent moment-load curve, with little variation from the regular curve for longer or shorter lengths of span. It becomes necessary then, in providing for the Mallet type, only to rectify the irregular kink in the Cooper uniform load curve. This gives a simple equivalent uniform moment-load diagram by which the Mallet or coupled locomotive is equally well provided for, with only a minor departure from the old standard *E*-loadings. By such means designers would relieve the much abused bridge computer of 90% of the tedious and tiresome work which is thrust upon him by the lack of judgment on the part of those responsible for the formulation of uselessly complicated railroad bridge loadings.

Part II.—Materials.†—Progress in the manufacture of steel has shown that a small percentage of copper tends to make the material rust resistant. Recognition of this fact is wanting in the Conference Committees' report.

Quality of Metal.—From the standpoint of safety and economy structural bridge steel should possess the greatest strength consistent with toughness. This quality is measured by the quantitative resistance of the material to applied force coupled with ultimate failure without brittleness.‡ In a $\frac{3}{8}$-in. plate of soft steel a $\frac{1}{8}$-in. hole may be enlarged by drifting to eight times its area, and a $\frac{3}{4}$-in. hole in a $1\frac{3}{8}$-in. bar may be enlarged to six times its area and bent double without cracking. These characteristics of soft steel are excelled greatly by lead in the ease by which a hole may be enlarged by drifting or the material doubled over upon itself, but it is not a characteristic of toughness nor of the severity of a test, because it disregards the quantity of the work expended in distortion.

Article 808.§—For carbon steel a close approach to the maximum toughness and economic strength for the tension member and the greatest resistance to plastic deformation in compression members is found in the grade of steel used in the Forth Bridge. Instead of soft metal (structural bridge steel of the Committees' specification) for tension members a strength was required of 67 200 lb. per sq. in. and a maximum of 73 900 lb. per sq. in., with not less than 20% elongation in 8 in., and a minimum of 76 160 and a maximum of 82 830 lb. per sq. in., with not less than 17% elongation for compression members.

* "Elasticity and Strength of Materials," Section V, p. 8.
† *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2667.
‡ *Engineering News*, April, 1895.
§ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2668.

*Article 809.**—The elastic range of stress for this grade of steel is much higher than for the 55 000 to 65 000-lb. structural steel of the Committees' specification (Table 2), and it involves an increase in cost of perhaps $1.00 per ton. This increase in the elastic range is approximately 25% for the tension member to 40% for the compression member. With the manganese tolerance of between 30 and 40 points, such steel is far tougher than the structural bridge steel of the American Railway Engineering Association specification advocated by the Conference Committees.

That there is no difficulty in fabricating was demonstrated in the construction of the Forth Bridge, and that it is damaged no more by punching appears to be not only the writer's opinion, but that of the late James Christie, M. Am. Soc. C. E.,† who may be quoted as follows:

"Extensive evidence is now on record indicating that steel of good quality, in bars of moderate thickness, and below or not much exceeding 80 000 lb. tensile strength, is not any more, and frequently not as much, injured as wrought iron by the process of punching or shearing."

Because columns of A. R. E. A. structural bridge steel in general develop less compressive strength than the wrought iron produced years ago, a tougher, stronger, and more suitable material is needed for bridge construction.

Article 1107.‡—The bending test recommended by the Committees as a criterion of the fitness or the unfitness of the metal is unscientific and inadequate. To explain this criticism it will be necessary to discuss briefly certain phases of the smelting of steel.

The melt is cleaned of harmful quantities of oxygen and oxides by the addition of certain elements having a stronger affinity for oxygen than iron at steel-making temperatures. The oxygen leaves the iron to combine with such elements, forming new products insoluble in the iron, which are largely precipitated and leave the metal.

Thus, the early product of the Bessemer converter was largely unfit for use because the metal poured from it was full of blow-holes. By adding "Spiegeleisen" containing 20% of manganese a fairly sound setting steel was produced. To insure certainty of desired results some excess of manganese over and above the minimum required is added in open-hearth practice as well as in the Bessemer. This excess, varying from 20 to 100 points, does not greatly affect the physical qualities of the softer grades of steel as indicated by ordinary methods of testing. It tends to vary the tensile strength an amount increasing with the carbon content. It apparently enables the specimen to withstand a cold-bend test better; but if a reverse bend is superposed upon the positive the effect of higher manganese in reducing the dependability of the material under compression becomes manifest. The rational limitations of manganese is given no consideration in the Committees' report. Manganese§ so reduces the plastic compressibility of the metal that a fracture in a bend test does not start on the tensile side of the specimen, as in carbon steel with a low manganese content, but on the compressive side instead.

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2668.
† *Transactions*, Am. Soc. C. E., Vol. XXX (1893), p. 160.
‡ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2675.
§ "Elasticity and Strength of Materials," Section V.

In bending with the hammer the specimen is seemingly tough as long as the hammer bends it in the same way, but a light blow in the reverse direction will snap the specimen as if it were glass.

Specimens of the wire first spun in the cables of the Mt. Hope and Detroit Bridges were bent around a small mandrel without fracture on the outside, but when they were bent around a 9½-in. pulley and then enlarged in a loop 17 in. in diameter, many of them cracked. A specified reverse bend test superposed on the positive would have avoided the delay and loss in both instances. The reverse bend should extend through an angle at least one-half as great as the positive bend.

This uncertainty* of the early Bessemer product illogically caused sentiment to gravitate toward the softer grades of open-hearth steel in place of a more economic intermediate grade used so effectively by the late Benjamin Baker, Hon. M. Am. Soc. C. E., for the Forth Bridge. Thus, the A. R. E. A. specification and that of the Conference Committees as well specify a grade of structural bridge steel capable of securing for the owner only about seventy cents worth of strength for a dollar's worth of metal.

Article 803.†—*Ladle Analysis.*—The tolerance for manganese in the ladle analysis should not exceed 35 points. The ladle analysis representing closely the average chemical composition of the melt may differ widely from the analysis of various parts of the solidified metal of the ingot as illustrated in a table presented by Talbot.‡

In that analysis the uniformity of the distribution of sulfur carbon and phosphorus effected by the small amount of aluminum is remarkable in view of the wide variation in the untreated ingot. The high-carbon, sulfur, and phosphorous content in the third drilling at the top of the untreated ingot exceeded a reasonable tolerance for sulfur and phosphorous by 400 per cent. This indicates undesirable physical properties and lack of dependability such as several years ago the writer found in some 6 by 6 by ¾-in. angles for the shoe of a railroad bridge. A sharp blow of an ordinary machinist's hammer was all that was required to break a piece out of the leg.

To disclose the worst metal in the ingot an analysis should be required from metal at the center of the plane of discard at the top of the ingot, and the limit of tolerance should be applied thereto.

Article 801.†—In the absence of specific limitations undesirable furnace practice may be encouraged. Thus, mild steel is frequently poured into the ingot while still boiling instead of being quieted by alloy treatment. Segregation and pipe are reduced by this procedure, but the resulting ingot is spongy and dependence is placed upon the weldability of the mild steel during rolling to remedy the unsoundness of the ingot. For the tension member flattening the gas-pockets and welding their sides more or less imperfectly is not so objectionable since the pull tends to press the welded surfaces together. Conversely, when the material is opposing resistance to compression these surfaces are subjected to a strain tending to pull them apart and the reduced

* A. E. Hunt, *Journal*, Iron and Steel Inst., Vol. II, 1890.

† *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2667.

‡ Harbord and Hall, "Metallurgy," Vol. I, p. 381; and *Journal*, Iron and Steel Inst., 1905, p. 204.

resistance of the metal compared to wrought iron in the column test seems otherwise unaccountable.

*Article 1003.**—Chromium, like manganese in small amounts, increases the hot ductility of the metal, raises the elastic ratio, and tends to make the hot surface less sensitive to quenching during rolling. This permits the use of a higher carbon content with safety in structural metal which is to be used untreated. A limitation of manganese content to 0.35% with carbon not exceeding 0.55%, silicon not more than 0.40%, and chromium, 0.15% and 0.20%, should secure an untreated steel of higher compressive resistance than structural nickel steel of standard specifications with 0.45% carbon and 0.70% manganese. It would present greater toughness in a reverse bend test.

With this specification as a base, the cost of silicon steel would be less than that of nickel steel since the silicon would cost little more than the manganese which it displaces, and the chromium would add to the cost of the metal only a fraction of the $40 extra per ton required for nickel steel.

By properly heat treating 3½% nickel steel a tensile strength of 170 000 lb. per sq. in. and a yield point of 135 000 lb. may readily be developed. By varying the heat-treatment quenching at 800° cent., the tensile strength runs more than 200 000 lb. and the elastic limit, 170 000 lb., with hardness drawn at 316° cent.; or, if the hardness were drawn at 538° cent., the tensile strength is approximately 140 000 and the elastic limit 120 000 lb. per sq. in. The metal thus treated should prove an economic material for eye-bar chains in long spans, advantageously embodying the toughness and dependability under high stresses that milder carbon steel develops under lower values; but there is no economic warrant for its use in its untreated state.

Conclusion.—In the desirable unification of bridge specifications the danger of standardized stagnation can be avoided only by the most searching and critical review of test methods of the past and a searching investigation of their significance and scope.

Economic waste may be avoided by a historical review of the studies and accomplishments of the past with full consideration of the advances in furnace practice in producing the most suitable material for the engineer's work.

Designers should establish working stresses upon the broad foundation of extensive investigations commencing with the early days of transition from wrought iron to steel. This was a period during which the fitness of the metal and its advantages over wrought iron was challenged and defended, sifting the rational from the irrational in the diversity of opinion developed by the most rigid application of the mechanical laws of elastic resistance.

GUNNI JEPPESEN,† M. AM. SOC. C. E. (by letter).‡—The writer desires to endorse and emphasize the following recommendations which have already been made—in part, or in full—in the course of this discussion:

1.—That the Conference Committees prepare a memorandum giving its reasons for the more important departures from present practice, embodied in these specifications.

2.—That a chapter covering erection be added.

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2673.

† Designing Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

‡ Received by the Secretary, August 22, 1930.

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* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2673.

† Designing Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

‡ Received by the Secretary, August 22, 1930.

3.—That the specifications be subdivided as follows: "General", "Design", "Detailing" (or "Details of Design"), "Materials", "Fabrication", "Weighing and Shipping", and "Erection".

4.—That a note be added calling attention to such items as may be covered by the contract of which the specifications are a part (for example, Article 7, "Patented Devices").*

5.—That Cooper's system of loading be retained.

6.—That a single system of unit stresses be used for dead and live loads, and that the difference in character of the two kinds of stress be taken care of in the impact formula only.

7.—That Article 302† be omitted.

The writer also recommends:

8.—That Articles 701 to 704,‡ inclusive, be rewritten in much greater detail and so that they may be read and applied literally with no "interpretation" whatever.

9.—That the Committees communicate with the Conference Committees on Steel Highway Bridges (American Association of State Highway Officials and the American Railway Engineering Association) and co-operate with them in obtaining uniformity between the two sets of specifications as far as possible, particularly with respect to: A single set of unit stresses; a single set of live loads for any one bridge; and identical methods of establishing pay weights.

Item 5.—There are weighty arguments against Cooper's system of loading, but in the writer's opinion they are offset by the facts: (*a*) That it has been the standard of comparison for a life time; (*b*) that it is known and understood by railroad men generally; and (*c*) that the large amount of estimating and design data which has been compiled and published on the basis of this system of loading would be made worthless by a change.

Item 6.—The use of two sets of unit stresses is cumbersome, if not entirely impractical. This is recognized in the parenthetical clause in Article 300§ which recommends that the live load stresses be increased by 50% (does this apply also to all other loads except dead load?) before being combined with the dead load stresses and that unit stresses 50% in excess of those contained in Article 301‖ be applied to the resulting combined stresses.

If this is done, use will never be made of the unit stresses, as listed in the specifications, which seems rather confusing, and the detailer will be unable to make use of standard handbooks as to rivet values, etc., and such accumulated data—based on present practice—as he may have at hand.

A better procedure would be to multiply the dead load stresses by two-thirds before combining them with the live load stresses and then to apply the specified live load unit stresses which are the same as those that have been in common use for the last twenty years.

Item 7.—Article 302 means nothing to the designer and, so far as can be seen, does not affect the design of the bridge in any way; but the executive is certain to interpret it to mean that the Engineering Profession considers 24 000 lb. per sq. in. a safe unit stress for a railway bridge.

* *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2649.
† *Loc. cit.*, p. 2655.
‡ *Loc. cit.*, p. 2666.
§ *Loc. cit.*, p. 2653.
‖ *Loc. cit.*, p. 2654.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion*

By Messrs. A. H. Finlay, R. F. Lyman, Jr., R. A. Caughey, Orrin H. Pilkey, and I. Oesterblom.

A. H. Finlay,† Assoc. M. Am. Soc. C. E. (by letter).‡—As the author states, a method of analysis is of value only if it is ultimately useful to the designer. The method of analysis so briefly and yet so fully described in Professor Cross' paper amply fulfills this condition, and structural engineers are fortunate in having it made available to them. Since 1922 Professor Cross has used in his classes the method of distributing unbalanced moments and, since 1924, when he developed the method in its present form, he has taught it to his graduate students with conspicuous success.

The authors of the many papers which have recently been published in connection with this general subject have interested themselves particularly in the analysis of continuous girders and have all approached the problem from more or less conventional directions—directions long since indicated by Clapeyron, Fidler, and others. It is probably sanguine to expect anything inherently new to result from such an approach. To Professor Cross must go the credit for abandoning the paths of his predecessors and for opening up a refreshingly new and eminently useful avenue of approach to the general problem of continuity.

Professor Cross himself states§ that the mathematically minded may object to solving a set of simultaneous equations by trial, which, of course, is what the successive approximations amount to. Such an objection from the mathematical standpoint is perfectly valid; indeed, it is the sort of objection which a mathematician would be certain to raise. From the standpoint of the designer, however, the trial solution constitutes one of the most important

* Discussion of the paper by Hardy Cross, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Asst. Prof. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

‡ Received by the Secretary, July 12, 1930.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 920.

points in favor of the method since, at any time, even after one distribution, the designer is able to form a good idea of the final values of the various moments. The values thus obtained may indicate a complete revision of the elastic properties of the structure. If this is the case he is then at liberty to make the necessary changes, and the only time lost is the negligible time required to make the first distribution. On the other hand, had the equations been set up no idea could have been formed of the final values until the solution had been completed. Only the experienced designer of indeterminate structures will appreciate the importance of this point. Between the analysis of a given structure, which is essentially mathematics, and the design of a required structure, which is essentially art, lie many difficulties.

As the author states the adopted sign convention permits the retention of the usual convention regarding positive and negative moments in the girders. By an ingenious twisting of the sheet this convention is stretched to fit the columns. So far, so good. The writer's criticism is that this sign convention makes it necessary, first, to distinguish between members on the right of a joint and members on the left of the same joint (this distinction when columns are involved is purely arbitrary and may well lead to confusion in complicated structures); second, it requires during the distribution of the unbalanced moment at a joint, that those portions of the unbalanced moment which are distributed to members on one side of a joint be given a different sign from those portions distributed to members on the other side of the same joint. This requires unnecessary thought during the distribution and may well be a contributory source of error. These two characteristics are inevitably associated with any convention based upon retaining that used by the author with regard to positive and negative girder moments. The question arises, is this retention worth while? Professor Cross states that, in his opinion, it is. With due deference the writer feels that it is not and suggests the following: Any moment which tends to rotate any joint clockwise is to be considered positive; the reverse, negative.

As an illustration of the application of this latter method consider Joint C in Professor Cross' example (Fig. 1*). In the first instance the unbalanced moment will be —30, being the algebraic sum of the fixed-end moments at the joint in accordance with the writer's convention. In other words, the locked joint is tending to turn counter-clockwise and will so turn when released. This unbalanced moment is distributed exactly as in Professor Cross' example with the important exception that all distributed portions have the same sign—in this case, plus. Consideration will show that, in all cases, the sign given to the distributed portions of the unbalanced moment is opposite to the sign of the unbalanced moment. It is thus apparent that there is no need to consider on which side of a joint a member occurs; and thus an arbitrary distinction is avoided.

If this sign convention is adopted a little consideration will show that the carry-over factor becomes $+\frac{1}{2}$ instead of $-\frac{1}{2}$, as used by Professor Cross. In other words, one-half the distributed moments are carried over to adjacent joints as before, but the signs remain unchanged.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 922.

In order to clarify the foregoing, Fig. 6 has been drawn to show the distribution of moments in the frame used by Professor Cross (Fig. 1), in accordance with the writer's sign convention. Selecting the member, *C-D*, Fig. 6 shows that the final value of the bending moment at the end, *C*, is + 257.2; that is, the member is tending to turn Joint *C* clockwise. It is evident that tension must exist on the top of the member, *C-D*, at Joint *C*. The signs of all other moments in the frame may be similarly translated.

Professor Cross has wisely restricted the paper to the bare description of the method. Those who use his method no doubt have their own particular procedure or technique developed in practice or teaching; indeed, if such is the case it is a healthy sign. The writer, who has used the method both in teaching and in practical design since 1926, has found himself less likely to make errors if, as soon as the moments are distributed at any joint, the correct proportions are at once carried over to adjacent joints before any further distributions are made. Often this procedure is of advantage in so far as the carried-over moments may reduce any unbalance that previously existed at such joints, thereby hastening the convergence.

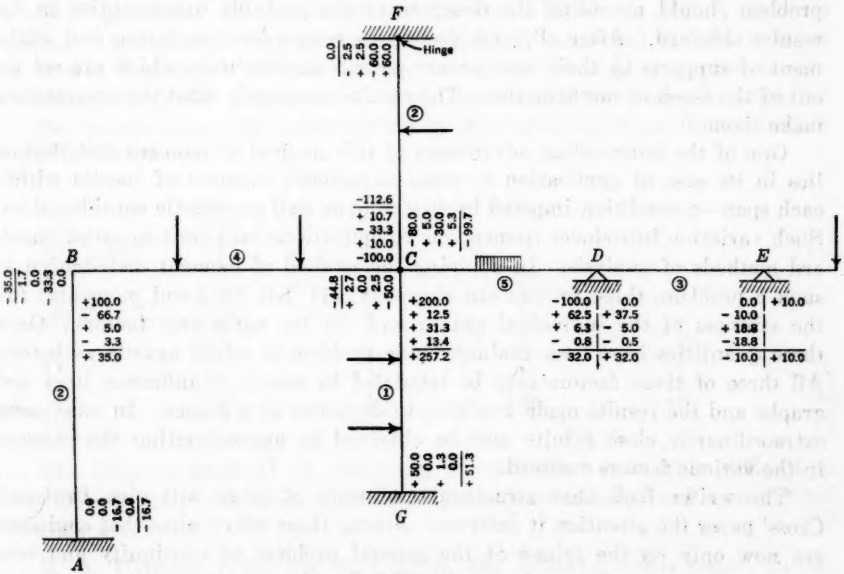

FIG. 6.

In practice it will be found quicker, and generally more advantageous, to treat those members having a hinge at one end as the paper suggested, by recognizing the existence of the hinge from the outset. As Professor Cross states, the stiffness of such a member must be modified in keeping with existing restraints, and it must be remembered that no moment can be carried over to a hinged end. It is perhaps hardly necessary to point out that the fixed-end moment for such a member is that obtaining when one end is hinged.

The speed of the method should be at once apparent. Assuming rollers at the support, D (Fig. 1), and a pin support at E, the frame is eight times statically indeterminate with respect to the external reactions, and yet the solution requires practically no effort.

The application of the principle of distribution to joint forces as well as moments* opens up an avenue of great importance in the problem of continuous arches on elastic piers. The further complication introduced by side-sway may be easily overcome, as suggested in the paper.* In the writer's opinion side-sway is seldom of importance in continuous viaducts of three spans and may be ignored in all ordinary cases where more than three spans are involved.

Moments induced by temperature changes and settling supports may be handled conveniently by the method. The procedure will be apparent when it is realized that the fixed-end moments from such sources may be readily calculated for any assumed temperature variation and settlements. These moments will result, in general, in unbalanced moments at the joints which may be distributed in the usual way. However, the facility of treating this type of problem should not blind the designer to the probable uncertainties in the results obtained. After all, such factors as temperature variation and settlement of supports by their very nature possess uncertainties which are set far out of the reach of mathematics. The results are merely what the assumptions make them.

One of the outstanding advantages of this method of moment distribution lies in its ease of application to cases of variable moment of inertia within each span—a condition imposed by economic as well as esthetic considerations. Such variation introduces tremendous complications into most so-called standard methods of analysis. In applying the method of moment distribution to such a problem, three factors are changed: (1) The fixed-end moments; (2) the stiffness of the individual spans; and (3) the carry-over factors. Once these quantities have been evaluated, the problem is solved exactly as before. All three of these factors may be tabulated by means of influence lines and graphs and the results made available to designers at a glance. In most cases extraordinarily close results may be obtained by approximating the changes in the various factors outlined.

The writer feels that structural engineers at large will give Professor Cross' paper the attention it deserves. Among those who realize that engineers are now only on the fringe of the general problem of continuity and who welcome each new advance, his work will find enthusiastic support such as may even overcome the inertia of things as they are.

R. F. Lyman, Jr.,† Assoc. M. Am. Soc. C. E. (by letter).‡—In the practical application of the principles of mechanics to the analysis of a rigid frame, the designer is confronted with several problems, some of which are quite often difficult to solve. Among these may be mentioned the following:

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 927.
† Asst. Engr., H. J. Brunnier, San Francisco, Calif.
‡ Received by the Secretary, August 8, 1930.

(1) The relative stiffness of the various members of the frame, which, in a design problem, must necessarily be estimated in advance.
(2) The value of the connections, whether fixed, hinged, or partly restrained, as, for example, the end condition at the juncture of column and footing.
(3) The extent to which unbalanced loading should be considered as a factor in the design.
(4) The determination of the moments, shears, and direct stresses in the members of the frame.

In reinforced concrete design Problem (1) is further complicated by the varying moment of inertia of some or all of the individual members.

The method described by Professor Cross provides a solution of Problem (4) that is simple enough to be used, within practical limits of time and effort, in the design of the more complicated types of frames. The writer prefers graphical methods where they can be used, but he has not found such methods feasible for any but the simplest types of frames.

The writer believes that the subject of side-sway due to dissymmetry or transverse loading is of considerable importance. For example, in the frame shown in Fig. 7 the correction of the moment at *A* was found to amount to about 23 per cent. In the analysis of the frame shown in Fig. 8, which is typical of grandstand and stadium construction, it was found that the moment at *D* was very much affected by the correction for side-sway, due to dissymmetry, when the deck was uniformly loaded. Furthermore, the moments at other points are considerably affected by this correction when the span, *C-D*, alone is loaded.

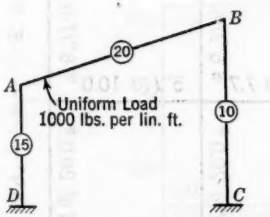

FIG. 7.

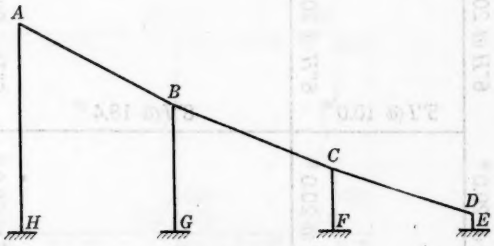

FIG. 8.

The author's method* of correcting for side-sway is a little difficult to understand. It could, with profit, be explained more fully, and a numerical example given.

R. A. CAUGHEY,† M. AM. SOC. C. E. (by letter).‡—This paper, in the writer's opinion, covers very important ground. As time goes on, and as people become more accustomed to using it, the value of the method described will be much more appreciated for use in problems, simple and complex.

One of the advantages of the method is that the application does not require complex mathematics; it requires nothing more than simple arithmetic. Another advantage is found in the time saved as compared with other methods; and still another is the automatic check of results.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 926.
† Associate Prof. of Structural Eng., Iowa State Coll., Ames, Iowa.
‡ Received by the Secretary, August 13, 1930.

The writer has presented to a class of undergraduate students, a problem involving a continuous beam of three spans with as many different moments of inertia and span lengths. After a brief presentation of the Cross method of analysis, supplemented by mimeographed notes, the problem was solved with practically no difficulty. The time involved was much less than the same class would require on the same problem by any other method known to the writer.

Mr. H. B. Gotaas, a graduate student working under the writer's supervision, computed moments in the members of a building frame loaded with vertical loads placed in various positions. Fig. 9 shows the dimensions and sizes of members in the bents of this building frame, which was actually built and used at the Iowa State College to compare stress measurements obtained by strain gauges to stresses computed by theoretic methods. On this project, results obtained by the slope deflection method (ordinarily considered the most accurate of theoretic methods), were checked against those obtained by the method of Professor Cross, and it was found that the maximum variation in unit stresses, considering complete sets of stresses for ten loading conditions, was only about 400 lb. per sq. in. It is also interesting to note that the time necessary to compute moments in a bent by slope deflection was about 25 hours for each loading, while that required by the Cross method was 4 hours. It will be agreed by all that the check on stress values is almost inconceivably accurate and that the time involved is reduced to allowable limits.

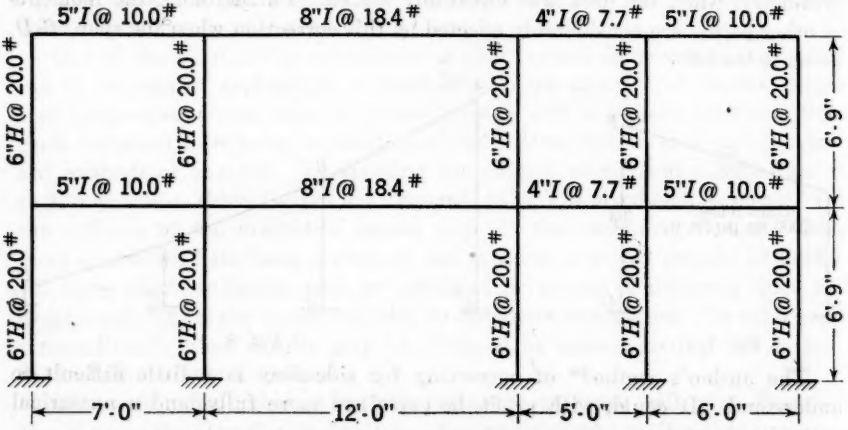

FIG. 9.—DIAGRAM OF BUILDING FRAME CONSTRUCTED FOR TEST PURPOSES AT IOWA STATE COLLEGE.

It might be well to state that in this comparison no correction was made for horizontal movements of joints. Had the frame been of great height as compared to width, this might have been necessary; but evidently it is not required in ordinary cases.

With concrete structures which are almost always constructed as rigid frames, with the welding of steel structures developing rapidly, and with an increasing desire on the part of designers to obtain stresses more accurately and to use material more economically, the writer prophesies a wide use for this method.

ORRIN H. PILKEY,* ESQ. (by letter).†—The Cross method of distributing moments is especially valuable in that it keeps intact the picture of the action of the structure. The user does not become lost in a maze of mathematics, rules, or formulas. However, the paper is entirely too brief. Some phases upon which entire chapters could be written with profit, are passed over with a concise statement.

For instance, the discussion of multi-storied bents subject to side-sway is limited to two short paragraphs.‡ The writer feels that this is not sufficient, especially to one unfamiliar with moment distribution. He agrees with the statement‡ that, except for the effect of transverse loads, the joints may be considered fixed in position in practically all rectangular frames. It should be recognized, however, that cases will occur in practice in which it will be important to make certain whether or not the effect of movement in the joints is appreciable. The following example will illustrate in detail a method of making the necessary correction.

The bent in Fig. 10 presents a rather extreme case of loading, but even in this case the movement of joints is so small that the refinement obtained by correction for it is mostly of academic interest. The percentage of error at some of the joints is certainly great, but the actual magnitude of the error is negligible. It should be noted that it is the magnitude of the error rather than the percentage that is significant. For unsymmetrical loadings encountered in most buildings the movement of joints is so small as to affect the results very little.

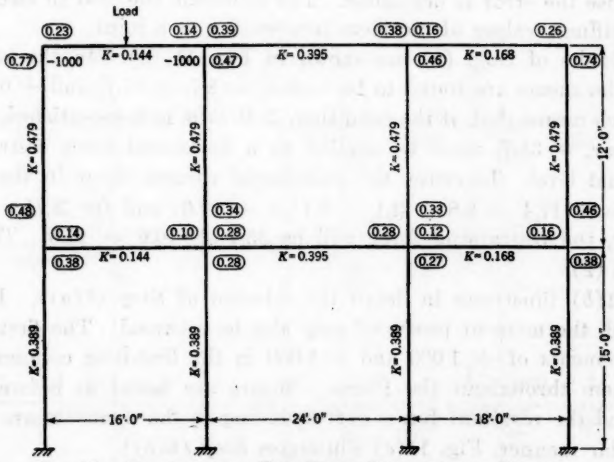

FIG. 10.—DIAGRAM OF A TWO-STORY BENT WITH A VERTICAL UNSYMMETRICAL LOAD.

Another point, illustrated by Fig. 10, is that the distribution of fixed-end moments is not practicable for analyzing multi-storied buildings subjected to appreciable side-sway, such as would be caused by wind pressure or other transverse loads. It would be more advantageous to distribute joint rotations.

* With Waddell & Hardesty, New York, N. Y.

† Received by the Secretary, August 14, 1930.

‡ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 927.

Outline of Steps in Determining Moments Caused by Side-Sway.—Specifically, the method of correcting for side-sway may be conveniently elaborated and described in the following successive steps:

(1) Assume that the joints do not move sidewise and distribute moments;
(2) Find the magnitude of the transverse forces necessary to keep the joints from moving by determining the difference in the shears in the columns;
(3) Find moments in the frame as a result of these forces, by first subjecting each floor, successively, to an assumed load as follows:
  (*a*) Apply a horizontal force to the first-story beams so as to give any desired set of moments in the lower columns, and distribute.
  (*b*) Assume the first floor to be fixed, apply any force to the second floor-beams, and distribute once more.
  (*c*) Write simultaneous equations to find the correct proportion of the moments caused by the assumed forces necessary to hold the joints fixed.
(4) The results are the moments caused by the unbalanced shear and are to be added algebraically to the moments found by assuming the joints held against side-sway.

Example.—In Fig. 10 the load is unsymmetrical and causes the joints to move sidewise. For an exact answer the effect of this movement must be determined separately because, by moment distribution, the joints are assumed to be held against side-sway. Actually, this correction is generally not necessary because the error is negligible. The numerals enclosed in circles are the relative stiffness values of members intersecting at a joint.

The results of Step (1) are shown in Fig. 11 (*a*). In the second-story columns the shears are found to be -85.6, -8.7, $+57.7$, and $+0.9$, respectively, which means that, if the condition, $\Sigma H = 0$, is to be satisfied, the unbalanced shear, -35.7, must be applied as a horizontal force restraining the bent at that level. Likewise, the unbalanced column shear in the first-story columns is $+17.4 - 8.8 + 2.1 - 0.1 = +10.6$; and for $\Sigma H = 0$, at the first story, the restraining force will be $35.7 + 10.6 = 46.3$. This constitutes Step (2).

Fig. 11(*b*) illustrates in detail the solution of Step (3(*a*)). If the load is assumed, the moment produced may also be assumed. The first step is to assume moments of $+1\,000$ and $-1\,000$ in the first-floor columns and distribute them throughout the frame. Shears are found as before (see Fig. 11(*a*)), and the resultant forces corresponding to the moments are computed. In a similar manner, Fig. 11(*c*) illustrates Step (3(*b*)).

For this example, the equations specified in Step (4) are as follows:

$$+142.3\ C_1 + 295.8\ C_2 = 35.7 \quad \ldots\ldots(1)$$
$$+560.3\ C_1 + 369.7\ C_2 = 46.3 \quad \ldots\ldots(2)$$

When Equations (1) and (2) are solved simultaneously the values of C_1 and C_2 are found to be 0.00445 and 0.119, respectively. The correct proportion of the moments caused by the assumed loads (see Fig. 11(*b*) and Fig. 11 (*c*)) are then added to the moments found by assuming no side-sway (Fig. 11(*a*)).

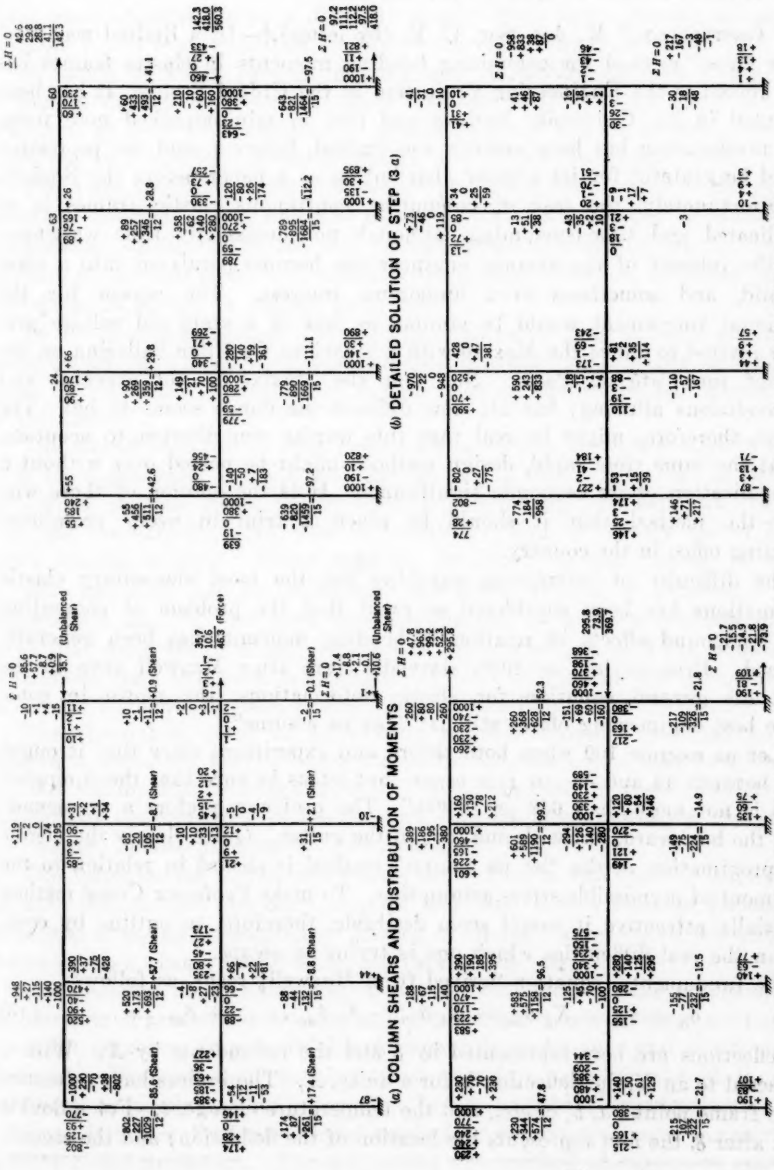
(a) COLUMN SHEARS AND DISTRIBUTION OF MOMENTS
(b) DETAILED SOLUTION OF STEP (3 b)
(b) DETAILED SOLUTION OF STEP (3 a)
(b) CORRECT MOMENTS: STEP (4)

FIG. 11.—COMPLETE SOLUTION OF PROBLEM TO FIND EFFECT OF MOVEMENT IN THE JOINTS.

The final correct results are shown in Fig. 11(*d*). A comparison between Fig. 11(*a*) and Fig. 11(*d*) will show that the error due to ignoring the effect of sidesway in this example is 27, or 3.4 per cent.

I. OESTERBLOM,* M. AM. SOC. C. E. (by letter).†—In a limited way Professor Cross' method for computing bending moments in elastic frames has been known to the Engineering Profession in the United States. It has been presented in his University lectures and also in mimeographed note form. This presentation has been entirely too limited, however, and the profession should be grateful for its greater distribution as a paper before the Society.

Unfortunately, the task of computing continuous elastic frames is so complicated and time-consuming—although not necessarily time wasting—that the interest of the average engineer has become paralyzed into a state of mild, and sometimes even humorous, interest. The reason for the occasional amusement would be similar to that of a staid old college professor invited to dance the Maxixe with a vivacious Castilian ballerina on the floor of the Café de Paris. Not that the Maxixe is not graceful and the conditions alluring; but oh, how difficult the dance seems to be! The danger, therefore, might be real that this worthy contribution to accurate, and at the same time rapid, design methods might be passed over without a full realization of its economic significance. It is the opinion of those who know the method that it should be given a trial in every prominent designing office in the country.

The difficulty of calculating anything but the most elementary elastic deformations has been considered so great that the problem of computing their compound effects in relation to bending moments has been generally shunned. Even as late as 1930, sixty-six years after Maxwell gave to the world his general equation for elastic deformations, the motto in some of the best engineering offices still is: "Let us assume".

"Let us assume 100 when both theory and experiment show that it ought to be between 45 and 55—or *vice versa*—but let us be sure that the computed stress is not more than 650 or 16 000." The professor prefers a promenade along the boulevard, but with one foot in the gutter. Quite clearly the degree of approximation in the "let us assume" method is absurd in relation to the refinement of permissible stress assumption. To make Professor Cross' method potentially attractive it would seem desirable, therefore, to outline by comparison the real difficulties which one is trying to escape.

The fundamental equation derived from Maxwell‡ reads, as follows:

$$\delta_a = \delta_{ao} - X_a\,\delta_{aa} - X_b\,\delta_{ab} - X_c\,\delta_{ac} \cdots\cdots + \delta_{at} \cdots\cdots\cdots\cdots (3)$$

Deflections are here represented by δ and the redundants by X. With a δ adjacent to an X, the deflection is for a unity, X. The indices have reference to the frame points, *a*, *b*, *c*, *etc.*, and the temperature change, *t*. For a double index after δ, the first represents the location of the deflection; and the second,

* Civ. Engr., Chicago, Ill.

† Received by the Secretary, August 18, 1930.

‡ This equation in some form occurs in all the textbooks and handbooks of Continental Europe; see, also, "Statically Indeterminate Stresses," by Parcel and Maney, New York, 1926, p. 112.

the location of its static or redundant cause. The symbol, δ_{ao}, signifies the deflection at a, with all redundants equal to zero. The equation represents the elementary fact that the deflection at any point is the sum of partial deflections, each attributable to a different cause. This is by the law of superposition, which is substantially true below the elastic limit of stress.

It should be noted further that for each redundant there is a sequence of partial deflections and that, therefore, one may write as many similar equations as there are redundants. Assuming every δ to be known, it would, therefore, be a simple problem of algebra to solve for the redundants.

There are two difficulties, however, to prevent a more common use of the universal method: First, one has to cope with the frequency of numerous redundants, which makes the use of determinants most necessary for the solution of the equations; and, second, each partial deflection, although purely a problem of statics, is difficult to ascertain.

To the average engineer the necessity for using determinants occurs so infrequently that he is very likely to get confused in his plus and minus signs. He is afraid, therefore, of the accuracy of the results and also afraid of the method.

The second objection is even more serious. There are many ways to compute the deflections: (1) By the equation of the elastic line; (2) by Mohr's method of moment areas; (3) by Castigliano's method of least work; (4) by equating the external work with the resilience; and (5) by the use of Maxwell's elastic weights. These methods are more or less similar or comparable and each has its advantages for given frames or elements. The unfortunate fact is that each partial deflection is quite a problem of its own; that there are as many* of them as the second power of the number of redundants; and that each is generally complicated through the compounding of several elements in more or less complex relations.

The elementary labor of integrating and summarizing each partial deflection is, therefore, very considerable, and multiplication of work to allow for all the redundants, together with the accumulation of these results into appropriate determinants, makes the task commercially impossible, except for very heavy, important, dangerous structures.

The formulas for elementary frames are generally well known; some complicated frames have also been analyzed and reduced to a series of formulas,† but many of the more complicated frames, or simple frames with a few unusual features, must always be solved as individual problems.

The question then arises: Should not the continuous frame be abandoned for the sake of greater safety and simplicity? Obviously not; there are numerous most necessary frames which are inherently continuous and can be nothing else. They can not be abandoned therefore; neither should they be analyzed, as loosely as they often are, "by the method of assumption".

Here is where Professor Cross' method is certain to render conspicuous service to the profession. Recently, using this method of successive approxi-

* By the use of Maxwell's law of reciprocal deflections the number of problems due to unity loads may be reduced by one-half.

† See, for instance, Kleinlogel "Rahmenformeln" and "Mehrstielige Rahmen," Berlin, 1925 and 1924.

mations, the writer checked a heavily loaded three-post reinforced concrete bent in 3 hours and with an error of less than 3 per cent. It was necessary to distribute and "carry over" only three times, whereas a very competent engineer had required several days to analyze it.

Knowing the universal method and its complications one is led to consider such a saving of time almost incredible and must necessarily ask in what manner such a miracle is possible. Professor Cross' method then appears to resolve itself into an interpretation and simplification of the universal method—and none can gainsay that it is both original and graceful. The reasons for simplicity and saving of time are outlined as follows:

(1) The very annoying product, *EI*, which is scattered all over the calculation sheets of the universal method has been combined with spans and heights into a "stiffness factor" on the basis of which any moment effect from whatever cause at any point a distribution is made in proportion to resultant resisting moments of adjacent elements. This eliminates in one stroke numerous petty notations and calculations of multiple occurrence.

(2) The travel of any moment induced at any point from any external source is then followed through along the lines of the frame by means of the "carry over factor" until such a point that the effect is dissipated to an insignificant magnitude. This means the elimination from the universal method of all those elements of expression, which merely denote an unnecessary refinement of calculation, ordinarily concealed under an innocent looking form, but none the less present as a burden on the calculations.

(3) Finally, there is the author's conventional grouping and summation, which takes the place—after all the "trimmings" have been eliminated—of the basic equations and their determinants.

In the analytical field Professor Cross' method takes a place with the method introduced in the graphical field by George E. Beggs, M. Am. Soc. C. E., of measuring the deflections of elastic models.* The two together form a very significant and admirable balance as against the profound theory of Continental Europe, which, excellent and extensively published, has none the less made little impression on the profession in the United States, except in designs of certain types of bridges and dams.

Professor Cross himself has confessed to a limitation of his method: It is not apparent how it could be applied, if the supports were truly movable. Perhaps this gap may be eliminated as experience is gained and more thought is given the subject. However, at present, this is of less importance, as most commercial problems are dealing with unyielding supports, and, for those which are not, it would be economically feasible to use the universal method at full length. These are problems that generally involve large quantities of costly construction materials, and sometimes also foundation difficulties, so that the cost of careful analysis is, therefore, not only small by proportion, but also most necessary to safeguard the heavy investment.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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THE TRAINING WALL ACROSS THE LIAO BAR IN MANCHURIA

Discussion*

By Messrs. Herbert Chatley, and H. C. Ripley.

Herbert Chatley,† Esq. (by letter).‡—The problem of the Liao Bar typifies conditions in North China and provides an interesting commentary on the question of single or double training walls. The regulation of the bar of the Whangpoo River presents some analogies. Before regulation this bar had a lowest low-water depth on the crest of 15 ft. Between 1907 and 1909 a training wall 5 500 ft. long was built on the left bank above half-tide level with a radius of about 7 000 ft., extending to the 36-ft. lowest low-water contour of the Yangtze River, of which the Whangpoo is a tributary. This wall reduced the natural "flare" of the mouth by about 40% and its concavity concentrated both flood and ebb, with the result that by 1912 the lowest low-water depth on the crest was increased to 21 ft., at which it remained stationary for three years. The lowest low-water sectional area of the stream above the training wall is about 42 500 sq. ft. The tidal influx varies from 400 000 000 to 5 000 000 000 cu. ft. and the efflux from 1 500 000 000 to 4 400 000 000 cu. ft. The non-tidal discharge averaged 12 600 cu. ft. per sec.

The material of the bar consists of Yangtze mud, about 90% passing a 200-mesh screen. There is about 10% colloid which makes the material fairly cohesive. In shallow areas coarser particles may be segregated, forming patches of fine, fairly clean sand, but this is not found in the channel. The water is charged with silt which is brought in by the flood tide from the Yangtze in quantities sometimes greater than 500 parts by weight per million.

In 1914 a second training wall, 7 000 ft. long, to half-tide level was built on the right shore parallel to, and 2 500 ft. from, the other and, from that date to 1928, the lowest low-water depth on the crest of the bar increased to 30½ ft.

* Discussion of the paper by P. N. Fawcett, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Engr.-in-Chf., Whangpoo Conservancy Board, Shanghai, China.

‡ Received by the Secretary, August 25, 1930.

Since then no appreciable change has occurred. This second wall reduced the low-water flare at the mouth by a further large amount, leaving only a taper of 1 in 100 between the walls. Parts of the areas between the walls and the old shore have since been reclaimed without appreciable effect on the bar.

While the conditions are very different from those of the Liao, the Whangpoo results show that the scour induced by a single concave wall can be usefully supplemented by the concentration of stream flow caused by a second wall.

H. C. RIPLEY,* M. AM. SOC. C. E. (by letter).†—This paper is a record, not of the method of improving the entrance to a harbor in the best and most economical way, but rather of securing the greatest channel depth at the entrance with the limited funds available. With this in view, the author seems to have met with considerable success.

The location and shape of the training wall are eminently suited for the purpose. The movement of material composing the bar at different stages of the tide is as follows: During flood the current crosses the wall at an angle, from the convex to the concave side, and by its overfall produces a deepening of the channel along the concave side and causes accretion on the convex side. During ebb exactly the opposite effects are produced until the tide has fallen sufficiently to uncover the top of the training wall. Then the wall becomes a real curved jetty and produces an effect as such. For the remaining two hours of the ebb, erosion takes place in the channel and the material thus excavated is carried across the channel and deposited on the convex side. Subsequently, a part of this material is washed back into the channel by the next returning ebb current, and the process is repeated by every recurring flood and ebb tide. However, the results have shown that the erosion in the channel is greater than the accretion; hence the beneficial effect of the training wall. The unfortunate feature of these effects is shown in the advance of the bar seaward, which will necessitate the continual extension of the training wall to keep up with the advance of the bar.

The author claims that no deepening of the channel is due to overfall during flood tide.‡ It is believed, however, that the builders of dams who have to provide against the undermining action at the base of weirs and others familiar with the results of overfall will be loath to accept the conclusions of the author in this regard.

If, on the other hand, the plan had been designed to make the best improvement regardless of any limit as to expenditure—except that necessary to carry out the plan—it would have been for a curved jetty instead of the curved training wall and the work would proceed as follows: A foundation would be built 3 ft. in thickness and of sufficient width to prevent settlement of that portion upon which the superstructure would be located for the full length of the proposed jetty from the shore to a point a short distance beyond the crest of the bar. Commencing at the outer end, the superstructure would be built toward the shore to a height of high water of the greatest spring

* Cons. Civ. Engr. (*Retired*), Brooklyn, N. Y.
† Received by the Secretary, August 27, 1930.
‡ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 938.

tide. In this way the advance of the bar would be prevented and no dredging would be required to develop the maximum channel that the curvature would permit.*

The cross-profile of the channel which a curved jetty will develop can be computed from the following formula:

$$Y = P\left(1 - \frac{X^2}{W^2}\right) + P\frac{26.28}{R}\left(1 - \frac{X^2}{W^2}\right)X \quad \ldots\ldots\ldots\ldots (1)$$

in which, P is the mean depth of the channel multiplied by 1.65; W is the half width of the channel; R is the radius of curvature of the jetty; and X and Y are the co-ordinates of the cross-profile, the origin being at the center of the channel at the surface of the water.

The value of P must be determined from the cross-section of the channel at the mouth when the river is at a high flood stage and the spring tide is at its maximum.

The navigability of the channel for the deeper-draft vessels entering the port may be increased to a limited extent by diminishing the width of the channel at the river, thus increasing the value of P and reducing the value of W.

A further enlargement of the channel if desired can be effected by dredging. This enlargement will be maintained by the jetty without additional cost for maintenance.

* See *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 1040, also Vol. 90 (1927), p. 207.

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PAPERS AND DISCUSSIONS

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EVAPORATION AS A FUNCTION OF INSOLATION

Discussion*

BY MESSRS. D. P. KRYNINE, R. C. WELLS, AND N. W. CUMMINGS.

D. P. KRYNINE,† M. AM. SOC. C. E. (by letter).‡—At first glance this paper seems to be very theoretical. Nevertheless, it must be considered as having a particular interest both for research men and practising engineers working in those fields of engineering connected with soils, water, and evaporation. The basic idea of the proposed methods seems to be correct, but for practical application, corrections corresponding to points of different longitude, due to local circumstances, must be studied further. Climatology and meteorology as applied to Civil Engineering is a broad field which has scarcely been touched. The writer has in mind one other work dealing with this subject from an engineering point of view.§

Determination of the Zenith Angle.—The author suggests determining the value of the zenith angle, ψ, by methods described by Bowditch.‖ The writer, however, recommends the equation:

$$\cos \psi = \sin \lambda \sin d + \cos \lambda \cos d \cos \omega t \ldots\ldots\ldots (15)\P$$

From this formula,¶ at a given time, t, the angle, ψ, may easily be found. The formula becomes very simple, if the observation is performed at noon. In this case the hour angle may be assumed equal to zero, and $\cos \psi = \sin \lambda \sin d + \cos \lambda \cos d$, or,

$$\psi = \lambda - d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The methods proposed by Mr. Bowditch are based on Equation (15) and require the use of "American Ephemeris and Nautical Almanac."

* Discussion on the paper by Burt Richardson, Esq., continued from September, 1930, *Proceedings*.

† Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

‡ Received by the Secretary, August 1, 1930.

§ "The Influence of Climate on the Building, Maintenance, and Use of Roads in the United States," by F. H. Eno, M. Am. Soc. C. E., *Proceedings*, Ninth Annual Meeting, Highway Research Board, December, 1929, p. 211.

‖ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 951.

¶ *Loc. cit.*, p. 958. The letter, d, corresponds to the declination of the sun; λ is the latitude; and ω, the angular velocity of the earth about its axis.

Determination of the Insolation Incident Upon the Exterior of the Earth's Atmosphere.—In Equation (15) the declination, d, is not constant during a given day, but changes gradually, being thus a function of time. Therefore, strictly speaking, the integration, Equation (12), must be considered as approximate. However, the first derivative of the declination with respect to time is small (less than 1′ of arc per hour), and introducing a mean value of d, as suggested by Mr. Richardson,* does not involve a serious error, especially since the coefficients, ρ' and F, probably, are sufficiently far from being accurate.

The writer regrets that only a few data are given in the paper with respect to the mean transmission coefficient, ρ', of the atmosphere at a particular place.

Research to Determine Values of the Coefficient, ρ; Determination of the Values of A.—The method of determining the coefficient, ρ, from Equation (2)† as proposed by the author,‡ is both simple and efficient. Actually, after using the pyrheliometer for determining ρ, the investigator does not need that instrument any more and may complete the remainder of the computations referring to evaporation without leaving his desk.

However, the difficulty with the method proposed by the author is due to the necessity of applying the coefficient, A, which represents the ratio of solar radiation to total radiation. The writer understands from Table 5‡ that the values of A are known only for a few localities.

For practical engineering purposes probably a rough estimation of the coefficient, A, is all that is necessary. Suppose the experiment is begun by taking the reading of a pyrheliometer corresponding to a zenith angle as close to zero as possible in given circumstances. The notation proposed is given in Table 14.

TABLE 14.—NOTATION FOR DETERMINING APPROXIMATE VALUES OF A_n.

Item.		Initial reading on the pyrheliometer.	Any subsequent reading.	Imaginary reading (zenith angle being equal to 0).
(1)	Zenith angle	ψ	ψ_n	ψ_0 (= 0)
(2)	Secant of the zenith angle	u	U	u_0 (= 1)
(3)	Value of the pyrheliometer reading, $\frac{dI}{dt}$	p	p_n	p_0
(4)	Correction coefficient	A	A_n	A_0

By substituting these symbols, Equation (3)‡ becomes:

$$p = \frac{K}{A} \times \frac{\rho^u}{u} \qquad (17)$$

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 958.
† *Loc. cit.*, p. 948.
‡ *Loc. cit.*, p. 951.

Suppose, now, that another pyrheliometer reading is taken at such a time that the secant of the new zenith angle is mu, in which, m is an arbitrary value. In this case, Equation (17) would be:

$$p_n = \frac{K}{A_n} \times \frac{\rho^{mu}}{m\,u} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Raise both sides of Equation (17) to the m power:

$$p^m = \left(\frac{K}{A}\right)^m \frac{\rho^{mu}}{u^m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

and solve simultaneously with Equation (18), thus:

$$\left(\frac{A\,p}{K}\right)^m u^m = \frac{A_n\,p_n}{K} \times m\,u \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Next, assume that the first reading on the pyrheliometer was made when the zenith angle was equal to zero. It may be possible that in the given locality such an angle may never be observed; but, nevertheless, Equation (20) may be applied to such an imaginary case, by substituting the values: $A = A_0$; $p = p_0$; $u = u_0$; and $m = U$. Then, Equation (20) becomes:

$$A_n = \left(\frac{A_0\,p_0}{K}\right)^U : \frac{p_n\,U}{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Equation (21) includes three unknowns, A_n, A_0, and p_0. In Table 5,* $A_0 = 0.87$ for $\psi = 0$ at all latitudes observed. Suppose the same condition occurs in the present case and that an approximate value of p_0 is:

$$p_0 = u\,p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Then a formula may be obtained by which to make a quick determination of the coefficient, A_n:

$$A_n = \left(\frac{0.87\,u\,p}{K}\right)^U : \frac{p_n\,U}{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which, K is the solar constant.

An examination of the function, A_n, and of its derivative with respect to U, will give an idea of the shape of the curve representing A_n.

Isolines of $f(u)$.—Let Equation (17) be written in the form:

$$p = \frac{f(u)}{A}$$

If the pyrheliometer's readings for a series of zenith angles are taken, the values of A may be easily computed. (See Equation (23).) Thus, in a given locality, all the values of $f(u)$ are also known. It is possible to represent $f(u)$ by drawing isolines for different zenith angles, within a certain territory. Then it would be possible to determine insolation and, consequently, the evaporation, at any day and at any locality within the area so plotted. The investigator must use the "Nautical Almanac", in order to determine the minimum zenith angle corresponding to the given day in a given locality and the times at which the zenith angles shown on the isoline maps take place. Interpolation may also be needed. This idea, naturally, is rough and is subject to improvement. The assumption is also made that $f(u)$ is not subject to considerable seasonal variations.

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 951.

The accurate determination of the coefficient, ρ, as such, is not essential for a civil engineer. It is possible, however, to estimate the value of ρ, even with a certain degree of accuracy, from the approximate pyrheliometer reading for the zero zenith angle:

$$\rho = \frac{p_0}{K} = \frac{p\ u}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Method of Measuring Evaporation.—The author suggests* computing evaporation, E, by means of Equation (10).

The value of B_r, in this equation is, in turn, given by a rather complicated formula, (Equation (8)†) and varies between 117 calories per sq. cm. per day to 148 calories per sq. cm. per day (Table 6†), the average value (weighted) being 131.4. Considering this average value as a 100%, the lower limit of B_r would be equal to 89% and the upper limit, to 113%, the deviation from the average being thus 11 to 13 per cent. Considering the fact that the back radiation—as a factor that prevents evaporation—may be evaluated as 23 to 35% (see Table 11),‡ its final influence on the result is between 2½ and 4½%, the average being about 3½ per cent. Since the result of the computation may be only approximate, it seems to be advisable to change Equation (10) somewhat as follows:

$$E = \frac{I - 131.4}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

or, in general:

$$E = \frac{I}{c} - c_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which, c and c_1 are constants.

Clearness Factors.—Personally, the writer is very much interested in the problem of determining clearness factors, since it may have some application not only in irrigation, but also to highway engineering. Certain road treatments involving the use of liquids ought to be different on sunny and shaded sections of a road. For instance, forest sections seem to need surface treatment different from field sections. The writer was led to these conclusions during an examination of the experimental road at Branford, for the Connecticut State Highway Department.

Obviously, the "heavy machinery" of zenith-angle determinations and integral calculus is not necessary in improving modest country roads, but there is a need for a rough field method of determining clearness factors to be used in such work.

R. C. WELLS,§ ESQ. (by letter).‖—This paper is of special interest because it approaches the subject of evaporation from a new point of view. It covers rather a wide range of geo-physical and meteorological measurements, and too

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 955.

† *Loc. cit.*, p. 953.

‡ *Loc. cit.*, p. 957.

§ Chf. Chemist, U. S. Geological Survey, Washington, D. C.

‖ Received by the Secretary, August 8, 1930. Published with permission of the Director of the U. S. Geological Survey.

much space would be required to discuss these in detail. Some of the measurements can be made with considerable accuracy, but others depend on natural conditions, such as clearness of the sky, the transmission coefficient of the atmosphere, etc., and, in view of the fact that some observations are incorporated in the author's calculations in the form of averages over limited periods, the agreement shown between calculated and observed evaporation is surprisingly good.

A reference* is made to the work of the writer concerning evaporation from Chesapeake Bay, from which it would appear that this was actually measured. This was not the case; the figures quoted by the author as the determined evaporation refer to the rainfall. In a case like that of Chesapeake Bay no check on the calculated evaporation is possible. On the other hand, when comparisons are made between observed and calculated evaporation for pans, the observed conditions are not generally what they would be over large bodies of water, so that it is not clear just what figures should be used in calculating the evaporation for the larger bodies of water.

The writer would be interested to have the notation† explained somewhat more fully for H, P_w, T_a, and T_w, especially the exact distances, or approximate distances, meant by the words "above the water" and "over the water". He agrees fully with the author concerning the great scarcity of reliable observations on evaporation, and is glad to see a definite number of ascertainable features listed for further measurement.

It is noted that in Table 11‡ the observed evaporation is lower than the lowest calculated, as well as higher than the highest calculated. Is this due to the use of a constant value for R, which is the ratio of convection to evaporation? Convection must express climatic conditions to a considerable extent and records for convection for different localities would be of interest. In fact, they are probably similar to those for evaporation; yet the figures in Table 7‡ show that convection is variable from hour to hour and from day to day, and R is there much more variable than E, the evaporation.

Another matter that perhaps should be discussed further before this problem is settled, is the bearing of the fact that insolation is practically zero at night whereas evaporation continues, and, in view of this fact, whether a variable that governs only when the sun shines furnishes as direct and as sound a basis for computing evaporation as some other that controls under all conditions.

The author has made a distinct contribution in tracing the heat gains and losses that may affect evaporation.

N. W. CUMMINGS,§ ESQ. (by letter).‖—It may be some time before the true value of Mr. Richardson's paper is fully appreciated, because economic pressure has not yet compelled the abandonment of unnecessary complications in estimating evaporation from lakes and reservoirs. For example, there are now

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 957.

† *Loc. cit.*, p. 946.

‡ *Loc. cit.*, p. 954.

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‖ Received by the Secretary, August 18, 1930.

various "standard" types of pan differing with respect to dimensions and method of installation. Those investigators who are accustomed to any one type are desirous of seeing that type perpetuated and its use extended so as to make readings at different times and places "comparable" with the ones with which they are already acquainted.

The manner in which any type of pan is to be made comparable to any lake is a mystery which many writers avoid mentioning, although it is difficult to see what benefit can be derived from making various pans comparable to each other unless they are comparable to lakes.

The late R. B. Sleight, Assoc. M. Am. Soc. C. E., found* that the ratio of the depth of evaporation from a pan 1 ft. in diameter is 155, while that from a pan 2 ft. in diameter is 129, and that from one 3.39 ft. in diameter is 121, on the assumption that the value for a pan 12 ft. in diameter is 100. The complete explanation of this variation will not be attempted here, but heat leakage through the walls undoubtedly contributes largely to it, as is shown by the fact that the divergence vanishes almost entirely when thermally insulated vessels are used. This is proved by the results of observations made at the Scripps Institution of Oceanography, University of California, from September 11 to December 23, 1929. During this interval of time many varieties of weather were experienced.

A thermally insulated pan, 2 ft. square and 8 in. deep, and a 1-gal. vacuum-insulated jar were run side by side. The diameter of the mouth of the jar was 6 in.; yet the depth of loss from the two vessels differed by only 1.5 per cent. Furthermore, the correlation coefficient between accumulated departures of evaporation and accumulated departures of insolation was 0.91 ± 0.01; while the corresponding coefficient for evaporation and wind was 0.01 ± 0.07.

These experiments show that important simplifications result from insulating the pan thermally and basing evaporation calculations on energy considerations, because difficulties concerning dimensions and manner of exposure are either eliminated or reduced materially.

* *Journal of Agricultural Research*, Vol. 10, No. 5, July 30, 1917, pp. 209–261.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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HIGHWAYS AS ELEMENTS OF TRANSPORTATION

Discussion*

By Messrs. W. W. Crosby, Walter Loring Webb, and Jay Downer.

W. W. Crosby,† M. Am. Soc. C. E. (by letter).‡—The writer has endeavored in the past to point out the fundamental significance of the proper location of highways.§ Mr. Lavis adduces an outstanding example of the sums involved, and the general interest therein may be proportional to their magnitude.

The author states‖ that,

"For handling large volumes of traffic through congested areas, highways should be designed [and paid for] for the main purpose of transportation rather than as adjuncts of the abutting properties." * * * "Where congestion of existing city streets interferes with the reasonably free passage of vehicles and pedestrians, highways of the type described are necessary not only to expedite through traffic, but also to restore such streets to their normal uses."

When these statements become generally appreciated, the highway will begin to be established in its proper place as an "element of transportation" comparable to the railway or canal.

It is true that a beginning has been made toward this end. The State highways of Pennsylvania have been largely located (or relocated) since 1924 with full recognition of the principles suggested by Mr. Lavis, although perhaps no single example of remarkable magnitude can be cited. Other States have attempted location more or less along similar lines.

However, the economist is not, and never will be, entirely free from the politician in this field. Perhaps this is just as well because the politician represents the sentimental side, and highways are not after all intended entirely for material purposes.

* This discussion (of the paper by Fred Lavis, M. Am. Soc. C. E., published in August, 1930, *Proceedings*), is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Coronado, Calif.

‡ Received by the Secretary, September 2, 1930.

§ *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1007.

‖ *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1361.

There are "intangibles" connected with any highway improvement that should not be overlooked even in its location. For example, one can estimate very closely the need for the width of a roadway to accommodate traffic, but to that width must be added, more or less arbitrarily, some further width for uses other than vehicular. Furthermore, it has been proved unwise to be too "economical" in that addition. Mr. Lavis refers to "Factors Other than Economic".* "A demand for a new type of road"† and a "Need of Special Highways"‡ are also suggested by the author.

At the International Road Congress at Milan, in 1926, the Italian delegates proposed the endorsement of the idea of building such special highways where traffic conditions might justify them (as in certain localities in Italy). They even suggested making them toll roads, provided the eventual return of such roads to the State was assured. The American delegates, probably fearful from their experiences with toll roads in the past, united with the British to block the passage of this resolution. However, this country is actually building similar roads now whether or not the toll factor is evident.

To criticize fairly the location of "Route 25", described by Mr. Lavis,‡ is impossible for the writer at present. Apparently, its alignment is questionably indirect, although Mr. Lavis seems to have anticipated this comment by his explanations. Probably here the "political" influence has swayed the economic decision. There seem to have been certain "legislative controls" and the influence of other route connections is mentioned as affecting the line. It is not quite clear, however, why the main route was swung aside to meet these other roads instead of bringing them to it. Neither is it clear to the writer why a roadway width of 50 ft. was selected. Do not most highway authorities condemn an odd number of lanes?

That "time is money" is now generally recognized by transportation authorities, and the author's exposition of the "Value of a Car-Minute" is commendable.§ His assumptions as to the division of the traffic on the highway are interesting, especially because of their apparent departure from the figures for New Jersey traffic reported in times past by the State Highway Commission. The writer believes that the assumed proportions‖ of heavy trucks, 50%, light trucks, 25%, and non-commercial vehicles, 25%, are almost the reverse of the earlier reported figures.

The author proceeds to use operating-cost figures that may be interestingly compared with those used in Pennsylvania.¶ He concludes his all too brief paper with a statement of "Modern Highway Demands".** The writer scarcely needs to reiterate now his position in this matter, but he does wish to felicitate Mr. Lavis on the forcible expression of his conclusions, which seem incontrovertible.

* *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1378.
† *Loc. cit.*, p. 1362.
‡ *Loc. cit.*, p. 1363.
§ *Loc. cit.*, p. 1368.
‖ *Loc. cit.*, p. 1369.
¶ "Highway Location," p. 134 *et seq.*
** *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1380.

WALTER LORING WEBB,* M. AM. SOC. C. E. (by letter).†—This paper demonstrates the possibility, and even the practical necessity, of applying the principles of economics to the construction of through highways, that new type brought into existence by the motor car—a highway designed to serve the traffic which desires to travel fifty miles or more without stopping and which is best served by avoiding all intermediate cities or obstructions of any kind. When the late Arthur M. Wellington, M. Am. Soc. C. E., brought out his book entitled "Economic Theory of the Location of Railways", he dealt with a subject which seemed very complicated to those who tried to master its principles, and yet Wellington had a subject which was comparatively simple. A railway has the right of eminent domain and can locate its line as it pleases; it has a private right of way on which no one else can travel; it does not stop its trains for highway crossings, and it is only a matter of finance to avoid grade crossings, and possible stops and delays, at intersections with other railways.

It is a comparatively simple matter to reconcile existing physical conditions with the conditions necessary for the economical movement of trains when a railway company has the broad powers of eminent domain and the absolute control of its own right of way, but the problem, as exemplified by the construction of Route 25, described by the author,‡ is infinitely more complicated. The State, several municipalities, and several railroad companies—each with their several powers of eminent domain and their several conflicting interests, each jealous of their powers to control matters to serve their own special interests—must be harmonized as nearly as possible. When one tries to introduce the added complication of designing a motor highway which will make possible the economical operation of motor vehicles, the problem becomes so complicated that it requires the highest grade of engineering skill and experience for its successful solution.

However, it should be realized that this last mentioned element—the economical operation of motor vehicles—is a vital and essential element in the solution of every such problem. In an address§ before the Engineer's Club of Philadelphia, Dr. George Otis Smith, Director of the U. S. Geological Survey, emphasized the necessity for the modern engineer to be an economist, or, in other words, that he must avoid all features of construction which necessitate methods of operation that are avoidably wasteful. In so far as the design for such a motor-highway project fails to provide for such economies of operation as are possible, to just that extent the design is a failure and is a waste of the several millions of dollars which are spent on it.

This subject is becoming increasingly important, since the construction of Route 25 in New Jersey is one of the first of many similar projects which will undoubtedly arise in the solution of the fast growing motor-transport industry.

* Cons. Engr., Philadelphia, Pa.
† Received by the Secretary, August 14, 1930.
‡ *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1363.
§ *Engineers and Engineering*, July, 1930.

The writer is aware that he has said little or nothing that has not already been well said by Mr. Lavis, but he believes that the matter is so important that it should be reiterated and emphasized until the general public, as represented by State and municipal authorities, who do not look at these matters from the engineer's standpoint, become more conscious of its importance as a matter of public welfare and economy.

It is unfortunate that the data on the actual cost of motor operation, the cost of applying brakes and resuming speed, the cost of stops and starts, and the effect of grade and curvature, are so meager. Undoubtedly, there will be an accumulation of such data in the near future. The necessity for the economical operation of motor-bus and motor-truck lines will probably result in the scientific study of those elements by the managers of those lines, and engineers will probably have, in the near future, a much large accumulation of data on which to base their calculations, than is now available.

Although the writer has no experimental data on the subject, he is of the opinion that curvature, even "easy" curvature, is not as harmless as it is commonly supposed to be. The centrifugal force which must be resisted in some way when a vehicle passes around a curve is measured by the formula, $C = \frac{W v^2}{g r}$. Its magnitude may be estimated by assuming numerical values as follows:

Let W = the weight of a car, or truck, assumed numerically as 15 000 lb.;
v = the velocity, assumed as 30 miles per hour, or 44 ft. per sec.;
g = the acceleration of gravity, or 32.2 ft. per sec. per sec.; and,
r = the radius of curvature, here assumed as 1 000 ft.

Then, C, the centrifugal force = 902 lb.

This stress must be taken up by the friction of the tires on the pavement, except as it would be possible to counteract it, at least partly, by elevating or banking the curve on the outside. In this numerical case, reducing the radius from 1 000 to 300 ft. and still maintaining the velocity at 30 miles per hour, would increase the stress from 902 to more than 3 000 lb.

It would probably be very difficult to determine experimentally the increase in rate of tire wear due to curvature. Part of the deterioration will be due to actual skidding or scraping of the tire on the pavement, thus wearing down the tire shoe; but some will be due to mere stress which, when long continued and repeated, will wear down the resistance even of the rubber, and cause its earlier destruction.

The objection to curvature may be very slight or even negligible, from the standpoint of loss of time or required increase in motive power—and hence in the cost of gasoline and oil—but the effect on tires and on the internal stresses in the car frame, which will cause more rapid deterioration, is certainly very appreciable, even if it is difficult to evaluate.

It is to be hoped that, when problems similar to those involved in the design of Route 25, are brought before the State or municipal authorities concerned, those elements of economy of operation will be thoroughly studied and will be given their due weight in making the final decision.

Of course, every such problem involves conflicting interests of greater or less importance. The public must insist that less important interests must give way (and perhaps be compensated) for a design which will best serve the interests of the majority. The danger lies in the possibility that powerful financial or political groups, whose real financial or public interest is less important, will be able to control the situation.

The profession owes much to Mr. Lavis for his practical demonstration of the possibility of the scientific application of Wellington's fundamental principles to such a complicated problem, and also for his demonstration that it is only by the use of such a general method that such problems can be properly solved.

JAY DOWNER,* M. AM. SOC. C. E. (by letter).†—This description of the Holland Tunnel-Elizabeth Highway known as Route 25, is a valuable exposition of the new conception of highway design and construction. In recent years marked improvements have been made in such component elements as grades, alignment, bridge structures, and paving materials. The outlook of planners and designers has now expanded beyond these primary essentials to include two other important elements of comprehensive design. These are the isolation of rights of way from abutting private property and the elimination of grade crossings.

The application of these last named principles in congested regions, such as that traversed by Route 25, even at very heavy costs, becomes justified when overbalanced by the economic pressure of losses due to traffic delays. The rapidly increasing volume of mechanical transport on roads in the United States has made evident, the analogous relation of the trunk-line, express highway to the railroad.

Through congested city sections, costly viaducts or tunnels are the principal means of accomplishing both isolation from abutting property and grade separations. In areas where vacant land is still available, this isolation as well as grade separation structures and ramps can be attained more economically by the acquisition of rights of way wide enough to afford flanking strips on both sides of the paved roadway.

This principle has been followed in laying out the Westchester County Parkways. The minimum width adopted for these parkways is 200 ft. and 300 ft. is desirable. The paved roadways which have been laid up to the present time are 40 ft. wide, but the Hutchinson River Parkway and the Saw Mill River Parkway have been sub-graded to a width of 60 ft. to provide for an additional 10-ft. lane on each side of the initial 40-ft. pavement.

The character of the greater part of the area traversed by Route 25 doubtless precluded the application of the excess right-of-way principle, but in laying out trunk-line express highways, there are doubtless broad fields offered for its application, even in the environs of many of the large cities.

Wherever it can be applied, this principle will doubtless commend itself to the designers of such highways, both for the economies in grade-crossing

* Chf. Engr., Westchester County Park Comm., Bronxville, N. Y.
† Received by the Secretary, September 11, 1930.

eliminations and the opportunities afforded for the amenities of parkway treatment applied to the flanking strips separating the paved roadway from abutting private property or subsidiary streets. The cost of excess land area may be easily outweighed by lower grade-separation costs and the very tangible economic as well as esthetic value of parkway treatment.

Through metropolitan areas of residential character, the effect of a trunk highway on near-by land values may be of greater economic importance to the immediate region than its value as a transportation route. The parkway type of express highway route invariably enhances near-by property values in contrast with the depression of values that has been experienced in many places along heavily traveled, modern roads in direct contact with private property.

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TESTS OF BROAD-CRESTED WEIRS

Discussion*

By H. W. King, M. Am. Soc. C. E.

H. W. King,† M. Am. Soc. C. E. (by letter).‡—Critical-depth devices for measuring flowing water are being used to a limited extent, particularly by the Bureau of Reclamation in the irrigated districts of the Western United States. The discharge has been considered to be a function of the up-stream head. For this method the theoretical discharge per foot of width of a rectangular cross-section is:

$$Q_1 = 3.087\ H_a^{\frac{3}{2}} \qquad (1)$$

Since H_a is the sum of the static head and velocity head the problem of correcting for velocity of approach is the same as for the weir. Moreover, all the tests in the paper and in the Appendix show that the coefficient varies considerably from the theoretical value of 3.087. Used in this manner the critical-depth device, therefore, does not provide a precise means of measuring flowing water. It is useful, however, for approximate measurements such as are satisfactory for apportioning irrigation water.

If a structure could be designed so that it would be known in advance that flow at critical depth would occur at a definite cross-section for all discharges, the discharge could be determined by measuring the critical depth, D_c, at this section and then making proper substitution in a simple rational formula. For a rectangular cross-section the discharge per foot of width is:

$$Q_1 = \sqrt{g}\ D_c^{\frac{3}{2}} \qquad (2)$$

With such a device there would be no coefficient to apply for velocity of approach, nor any other corrective factor to consider, and the measurement

* This discussion (of the paper by James G. Woodburn, Assoc. M. Am. Soc. C. E., published in September, 1930, *Proceedings*) is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

† Prof. of Hydr. Eng., Univ. of Michigan, Ann Arbor, Mich.

‡ Received by the Secretary, June 23, 1930.

of discharge would be reduced to its simplest terms. Another advantage is that it provides for measuring discharges with a minimum loss of head, as the tail-water can be nearly as high as the head-water without changing the position of critical depth.

If a channel is sufficiently constricted either by elevating the bottom, or by drawing in the sides, or by combining the two so as to form a short reach which has a gently sloping bottom, flow at critical depth will occur near the upper end of this slope. The position of critical depth can be predicted approximately, but not with the accuracy required if discharges are to be determined from measurements of this depth. Investigations of several types of structures by the Society's Special Committee on Irrigation Hydraulics, and by others, have failed to produce a design for which the position of critical depth could be predicted accurately.

Professor Woodburn began an investigation of this problem in the fall of 1928. His experiments on a broad-crested weir with rounded entrance and gently sloping crest showed that critical depths for the different discharges lie in quite regular lines down stream from the upper edge of the weir. The law affecting their position could not be determined. Since critical depths on this structure, as well as on other structures with which the writer is familiar, occur where the water surface has a comparatively steep slope, it is necessary that their positions should be known within very narrow limits. An error of ½ in. up stream or down stream in the point of measuring critical depth results in an error of several per cent. in the discharge determined by Equation (2).

An unsuccessful attempt was made to flatten the water surface near the point of critical depth by attaching an apron with a gentle slope to the up-stream end of the weir. Too flat a slope of the apron, by producing flow in the unstable condition existing near critical depth, causes waves to form on the weir crest with one or more points of critical depth occurring most irregularly. It appeared, therefore, that to assure the occurrence of critical depth at a fixed section very flat slopes on the weir crest should be eliminated.

Since Professor Woodburn's experiments with a single sloping crest showed critical depth to occur down stream from the upper edge of the weir, while with a level apron attached to the up-stream edge critical depth moved up stream and over the level apron, it appeared that the point of critical depth could doubtless be moved up stream or down stream by changing properly the respective slopes of the two aprons. Professor Webb's experiments were undertaken for the purpose of investigating this principle. (See Appendix I.)*

He was successful in finding slopes for the two aprons which caused the critical depths for all discharges to lie very close to vertical lines. It was found, however, that slight changes in the slope of either apron would displace points materially from these verticals and also that the position of the vertical when it was obtained was not precisely the same for the different combinations of slopes. One series of experiments in the Appendix shows the vertical to be over the apex; another shows it to be 0.08 ft. up stream from the apex; while a third indicates a position about midway between the other two. In view of

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1604, *et seq.*

the steep slope of water surface that obtains, if a device of this type is to be useful for precise measurement of water the position of critical depth must be known between much narrower limits than these.

The investigations of Professors Woodburn and Webb give much additional and useful information regarding flow at critical depth. They have not completely accomplished the original purpose of devising a structure which will produce critical depth at a predetermined cross-section. They have made progress, however, and by bringing to light conditions which affect the stability of flow as well as the position of critical depth, they have paved the way for intensive investigations in the future. In the writer's opinion it is quite probable that a design satisfactory for measuring flowing water by this method may yet be devised.

Incidentally, it is planned to continue these experiments on critical depth at the University of Michigan. It is hoped that similar investigations will be undertaken in other hydraulic laboratories.